Stochastic processes - written exam
June 2nd, 2020.

1. Consider two independent random variables X and Y. Probability density function of X is $\varphi_X(x) = \frac{1}{2}e^{-|x-1|}$, $x \in \mathbb{R}$, while $Y : \mathcal{U}(0, 1/2)$. Find autocovariance function of a stochastic process $Z_t = (t+1)XY^2$.

2. Due to covid-19 pandemic government decided to quarantine people and close borders. People are only allowed to go to supermarket, pharmacy or to visit a doctor if necessary and they can perform only one action per day. They can also stay at home all day. Maya is a chronic patient and she has to receive a therapy from time to time. Also, she will not perform the same action two days in a row. If Maya went to supermarket today, the next day she will go to pharmacy or to receive the therapy with equal probabilities, while she will stay at home with probability 0.6. If she received the therapy, the next day she will go to pharmacy. If she went to pharmacy, the next day she will go to supermarket with probability 0.4, while she will stay at home with probability 0.5. After all day at home, Maya will go to supermarket or to receive a therapy with probabilities 0.5. Find if, in the long run, Maya more often stays at home or goes to pharmacy.

3. Passengers arrive at a railway station according to a homogeneous Poisson process with intensity λ. At the beginning (time 0), there are no passengers at the station. The train departs at time t_s. Denote by W the total waiting time of all passengers arrived up to the departure of the train.

(a) Find W.

(b) Suppose that the number of passangers arrived at a railway station up to time t_s is 3. Find the expected total waiting time of all passengers arrived up to the departure of the train.

4. W_t is standard Brownian motion. Find

$$E(W_t^3 - 3W_tW_{\frac{t}{4}} + 3W_{\frac{t}{3}}W_{\frac{t}{4}} | \mathcal{W}_{\frac{t}{3}}), \quad t > 0.$$