Stochastic processes – written exam
18.6.2018.

1. On a multiple-choice exam with three possible answers for each of the five questions, what is the probability that a student would get four or more correct answers just by guessing?

2. Two gamblers play the following game. A fair coin is flipped. If the outcome is heads, player A pays player B 1 dollar, and if the outcome is tails player B pays player A 1 dollar. The game is continued until one of the players goes broke. Suppose that initially player A has 1 dollar and player B has 2 dollars, so a total of 3 dollars is up for grabs. Let X_n denote the number of dollars held by player A after n trials.

(a) Find the expected number of flips until one of the players goes broke.

(b) Show that X_n is a Markov chain and find the one-step transition probabilities.

(c) What is the probability that a game has stopped after only one flip of the coin?

(d) Find the two-step transition probabilities $P[X_{n+2} = i | X_n = i]$, $i = 1, 2$.

3. Satellites are launched into space at times distributed according to a Poisson process with rate 2 per year. A time that each satellite independently spends in space before falling to the ground is exponentially distributed with mean 40 years. Find the expected number of satellites on the ground after 20 years.

4. Find

$$E[\alpha W_t + W_t W_s + \beta W_r^2 W_s | \mathcal{W}_t], \qquad 0 < s \leq t < r,$$

if W_t, $t \geq 0$ is standard Brownian motion and $\mathcal{W}_t$ is history of Brownian motion until time t (including time t).

5. Consider a sequence of independent, identically distributed random variables

$$X_n : \begin{pmatrix} -6 & -2 & 2 & 6 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}.$$

Prove that $\gamma_n = \cos(\pi n) \sin\left(\frac{\pi}{2} S_n\right)$, $S_n = X_1 + \ldots + X_n$, $n \in \mathbb{N}$ is a martingale with respect to the filtration $\mathcal{F}_n = \mathcal{F}(X_1, \ldots, X_n)$.