Stochastic processes – written exam

April 1st, 2019.

1. Let X be the randomly chosen number from the set $\{1, 2, 3, 4, 5, 6\}$. Y is chosen from the same set among numbers that are integer multiple of X. Find $E(X|\mathcal{F}(Y))$.

2. Consider standard Brownian motion W_t. Discuss the value of

$$E(W_t^3 + 2W_s^2 + W_\alpha | \mathcal{W}_\alpha), \quad t > s \geq 0$$

with respect to a parameter $\alpha > 0$. $\mathcal{W}_\alpha$ is history of Brownian motion until time α (including α).

3. A Markov particle moves on points 0,1 and 2 arranged in a circle in the clockwise direction. A step in the clockwise direction occurs with probability p, $0 < p < 1$, and a step in the counter-clockwise direction occurs with probability $1 - p$. If Markov particle is initially positioned on point 0, what proportion of time the particle will spend on point 0? Explain.

4. Consider the emission of $\gamma-$photons by a radioactive source. This can be modeled to a close approximation by a nonhomogeneous Poisson process $X(t)$ with intensity function $\lambda(t)$ given by $\lambda(t) = \alpha e^{-\beta t}$, $t \geq 0$, $\alpha, \beta > 0$, where α is a parameter depending on the amount of the radioactive material and the β^{-1} is the mean life of the source. Calculate

$$\frac{\partial}{\partial s} P\{X(s+t) - X(t) = 0\} - \beta P\{X(s+t) - X(t) = 1\}.$$

5. Consider a sequence of independent integrable identically distributed random variables $Y_1, Y_2, \ldots$ having all positive values. Check if $X_n = (Y_1 Y_2 \cdot \ldots \cdot Y_n)^{1/n}$ is martingale with respect to filtration $\{\mathcal{F}_n\}$, where $\mathcal{F}_n = \mathcal{F}(Y_1, \ldots, Y_n)$.