Stochastic processes – written exam
16.4.2018.

1. Milan and Uroš play a dice game as follows. Each of them throws a dice, independently of the other one. If the sum is 5, 6 or 7, Milan wins. Otherwise, Uroš wins.

(a) Find a probability that Milan wins.

(b) What is the probability that Milan's throw resulted in 3, if it is knows that Milan won.

2. Suppose that Milan and Uroš play a series of games explained above. The overall winner is the first player to have won two more games that the other. Find the expected number of games played.

3. Consider a communications system which transmits the digits 0 and 1. Each digit transmitted must pass through several stages, at each of which there is a probability $p \in (0,1)$ that the digit entered will be unchanged when it leaves. Let X_n denote the digit entering the $n-$th stage.

(a) Find the state transition probability matrix.

(b) Find initial state vector such that the Markov chain $\{X_n,\, n \geq 0\}$ is stationary.

(c) Find $P\{X_{n+2} = 1 | X_n = 1\}$.

4. Customers arrive at the automatic teller machine in accordance with a Poisson process with rate 10 per hour. The amount of money withdrawn on each transaction is a random variable with mean \$40 and standard deviation \$50. A negative withdrawal means that money was deposited. The machine is in use for 16 hours daily. Approximate the probability that the total daily withdrawal is less that \$7000.

5. Find a constant a such that

$$V(t) = W^3(t) + aW^2(t) - 3tW(t) + t, \quad t \geq 0$$

is martingale, if $W(t)$ is standard Brownian motion.