Stochastic processes – written exam
1.2.2019.

1. Students are waiting for the test results. Teacher told them that the results will be visible in exactly 2 hours with probability $\frac{1}{2}$, and in exactly 6 hours with probability $\frac{1}{3}$. Otherwise, they will be able to see the results during the class that is going to start in 20 hours, and it will last for two hours (in that case the waiting time will be uniformly distributed). Consider continuous random variable X that is independent of students waiting time Y. Calculate $E(Y|X + \text{sgn}X)$.

2. Consider continuous and independent random variables X and Y such that $E(X^2) < \infty$ and $E(Y) = 0$. Find the mean of the stochastic process

$$Z_t = t(X^2Y + 1).$$

3. Peter decided to lose some weight and sometimes he will go to work by foot. Suppose that day is classified as rainy with probability p, independently of the previous day. Otherwise, the day is classified as a sunny. If the day is rainy, Peter will go to work by car, independently of the previous day. If the day is sunny, Peter decides to base his decision on the previous day. If on the previous day he went to work by car, and the day is sunny he will go to work by foot. If he went to work by foot, and the day is sunny it is equally likely that he will choose to go to work by car and by foot. What proportion of days will Peter go to work by car?

4. Consider babies born in the "normal" range of 37–43 weeks gestational age. Extensive data support the assumption that for such babies born in the United States, birth weight is normally distributed with mean $3432\,g$ and standard deviation $482\,g$. Also, assume that such babies are born at a Poisson rate of 20 babies per minute.

(a) What is the probability that the birth weight of a randomly selected baby of this type is between 3000 and $4000\,g$?

(b) Calculate the probability that in the next two minutes will be born 40 babies of this type with birth weight between 3000 and $4000\,g$. Find an average time between two consecutive "normal" births of babies.

5. Let X_t be a stochastic process with stationary and independent increments, zero expectation and variance $\sigma_X^2 t$. Prove that $Z_t = a(X_t^2 - \sigma_X^2 t) + Y_t$, $a \in \mathbb{R}$, is martingale with respect to the history of the stochastic process X_t until time t if Y_t is martingale with respect to the same filtration.