

# Stochastic processes – written exam

28.8.2019.

1. The plane loads 100 packages whose weights are independent random variables that are uniformly distributed between 5 and 53 pounds. What is the probability that the total weight will exceed 3000 pounds?
2. An absent-minded professor has two umbrellas that he uses when commuting from home to office and back. If it rains and umbrella is available in his location, he takes it. If it is not raining, he always forgets to take an umbrella. Suppose that it rains with probability  $p$  each time he commutes, independent of other times. In each unit of time professor has 0,1 or 2 available umbrellas at his current location. Suppose that professor initially had 2 umbrella at his current location. Find the probability of not having 2 umbrellas at the same location next day (i.e. after 2 commutes).
3. Suppose that taxis depart an airport at the Poisson rate of 2 per minute and that the probabilities of a taxi having one, two, or three passengers are 0.5, 0.3, and 0.2, respectively.
  - (a) Find the probability that number of taxis departing an airport in 10 minutes equals 10.
  - (b) Find the expected number of passengers departing the airport in 10 minutes.
4. Consider standard Brownian motion  $W_t$ .
  - (a) Prove that random variables  $X := W_{2s}^2 + W_s^2 + 3W_{2s} - 3W_s - 2W_{2s}W_s$  and  $Y := W_{\frac{1}{2}s}^3 - W_{\frac{1}{2}s}^2 + 2$  are independent for  $s > 0$ .
  - (b) Find  $E[E[X + Y|W_s]]$ .
5. Consider Poisson process  $N_t$  with rate  $\lambda$ . Determine if  $N^2(t) - \lambda t$  is martingale with respect to history of Poisson process.