STOCHASTIC PROCESSES - written exam **26.9.2017.**

1. One considers 100 people of whom 40 speak Russian, 30 English and 21 French. Then 15 speak Russian and English, 10 Russian and French, 5 French and English and 3 of them speak all three languages. Then one randomly chooses 3 of them.

(a) What is the probability that neither of 3 people speak foreign language?

(b) What is the probability that all three people speak Russian language?

(c) What is the probability that two of them speak some foreign language, but the third person speaks no foreign language?

2. Consider a standard Brownian motion W_t and define

$$X_t := W_t - tW_1,\ 0 \le t \le 1.$$

(a) Find a autocovariance function $K_X(t,s)$ for a given process X_t.

(b) Does the process $\{X_t\}_{0\le t\le 1}$ has an independent increaments?

3. Two girls and two boys are playing with the ball. Each boy will toss a ball to the other boy with probability $1/2$, and to each of the girls with probability $1/4$. Each girl will toss the ball to each boy with probability $1/2$ and won't toss the ball to the other girl. If the game last for a long time, how often will each of them receive the ball?

4. All the employees in one company use the same printer. Suppose that the times between two consecutive print requests are independent, identically distributed exponential random variables with parameter 10 per hour. The printer needs exactly 6 seconds to print one paper.

(a) What is the probability that exactly 20 print requests will arrive between $8:30$ i $10:30$h?

(b) What is the probability that a print request will arrive while the previous document containing 6 papers is not yet printed?

5. Consider a function $f : [0,\infty) \to \mathbb{R}$ and define a process

$$X_t := W_t^3 + f(t)W_t,\ t \ge 0,$$

where W_t is a standard Brownian motion. Determine a function f such that $\{X_t\}$ is a martingale.