WRITTEN EXAM – STOCHASTIC PROCESSES, 20.6.2017.

1. A batch of 25 injection-molded parts contains 5 that have suffered excessive shrinkage.

(*a*) If two parts are selected at random, and without replacement, what is the probability that the second part selected is one with excessive shrinkage?

(*b*) If three parts are selected at random, and without replacement, what is the probability that the third part selected is one with excessive shrinkage?

2. Let $(W_t)_{t\geq 0}$ be a standard Brownian motion. Find $E(W_4^2|W_2)$.

3. In working with a particular gene for fruit flies, geneticists classify an individual fruit fly as dominant, hybrid or recessive. In running an experiment, an individual fruit fly is crossed with a hybrid, then the offspring is crossed with a hybrid and so forth. The offspring in each generation are recorded as dominant, hybrid or recessive. The probabilities the offspring are dominant, hybrid or recessive depends only on the type of fruit fly the hybrid is crossed with rather than the genetic makeup of previous generations. The offspring of a dominant individual crossed with a hybrid are dominant 50% of the time and hybrid the other 50%. The offspring of a hybrid crossed with a hybrid are dominant 25%, hybrid 50% and recessive 25%, while the offspring of a recessive crossed with a hybrid are hybrid 50% and recessive 50%.

(*a*) Find the transition matrix for this problem.

(*b*) What is the probability the third generation offspring is dominant given the first generation offspring is recessive?

(*c*) If the population of fruit flies initially is 20% dominant, 50% hybrid and 30% recessive, what percentage of the population is dominant after 3 generations?

4. In good years, storms occur according to a Poisson process with rate 3 per unit time, while in other years they occur according to a Poisson process with rate 5 per unit time. Suppose next year will be a good year with probability 0.3. Let N_t denote the number of storms during the first t time units of next year.

(*a*) Find the probability that in the first t time units of next year have occurred n storms.

(*b*) Is $\{N_t,\, t\geq 0\}$ a Poisson process?

(*c*) Does $\{N_t,\, t\geq 0\}$ have stationary increments? Explain.

(*d*) If next year starts off with three storms by time $t=1$, what is the probability it is a good year?

5. Let $(X_n)_{n\in\mathbb{N}}$ be a sequence of independent random variables, such that $P\{X_n=1\}=p$ and $P\{X_n=-1\}=q$ for all $n\in\mathbb{N}$ and $0<p<1$, $q=1-p$. Define $S_n:=\sum_{j=1}^{n}X_j,\ n\geq 1$.

(a) Compute the probability mass function of the random variable $Y_i:=\left(\frac{q}{p}\right)^{X_i}$, $i\in\mathbb{N}$.

(b) Prove that the sequence of random variables

$$M_n:=\left(\frac{q}{p}\right)^{S_n}$$

is a martingale with respect to a filtration $\mathcal{F}_n=\sigma(X_j, 1\leq j\leq n),\ n\geq 1$. With $\sigma(X_j, 1\leq j\leq n),\ n\geq 1$ we denote σ-algebra generated by random variables $X_j, 1\leq j\leq n$.