WRITTEN EXAM – STOCHASTIC PROCESSES
January 25th, 2018.

*Explain your answers so that we know that you are not guessing.*
*What you don't write we have to assume that you don't know.*

**1.** Suppose that a couple after $n$ year of marriage can have at most $n$ children. Let $X$ denote a random variable which represents the number of children after exactly 3 years of marriage,

$$X : \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1/6 & 1/3 & 1/3 & 1/6 \end{pmatrix}.$$

If $Y$ is a number of female children after exactly 3 years of marriage, find $E(X|\mathcal{F}(Y))$.

**2.** Consider two independent random variable, $X$ and $Y$, where $X : \mathcal{E}(1)$ and

$$\varphi_{(X,Y)}(x,y) = \begin{cases} \frac{1}{2}e^{-x}, & x \geq 0,\, y \in (1,3), \\ 0, & otherwise. \end{cases}$$

Find autocovariance function of a stochastic process $N_t = X(tX + Y)$.

**3.** In some countries exists a limit on number of children the couple can have in marriage. Suppose that a limit is 4 i.e. a couple cannot have more that 4 children in marriage. Let $X_n$ denote the number of children in marriage after $n$ years. Suppose that the number of children cannot decrease. It is known that the number of children after one year will stay the same with probability 0.5, except in the case when there is 4 children. If in the end of one year the number of children is $k$, $k = 0, 1, 2$, then at the end of the next year the number of children will be $k + 1$ with probability $\frac{1}{k+2}$. If the couple has a child, the number of children will increase by 2 after 2 year with probability $1/4$.

($a$) Find a one-step transition probability matrix of a Markov chain $\{X_n\}$.

($b$) Is Markov chain $\{X_n\}$ ergodic?

**4.** The number of babies born is Serbia during $t$ days is a Poisson process. On an average, a baby is born after every 60 seconds. If it is known that in one of 10 cases baby is not discharged from the hospital after 5 days, find the probability that in two days period, the number of babies which are not discharged from the hospital after 5 days is at most 2.

**5.** Consider the standard Brownian motion $W(t)$, $t \geq 0$.

(a) Prove that for $s \geq t \geq 0$

$$E(W(s)W^n(t)) = \begin{cases} 0, & \text{if } n = 2k,\ k = 0, 1, 2, \ldots \\ n!!\, t^{\frac{n+1}{2}}, & \text{if } n = 2k-1,\ k = 1, 2, \ldots \end{cases}.$$

(b) Show that $E\big(|W(t) - W(s)|^2\big) = |t - s|$, $t, s \geq 0$.

*Hint:* Calculate $E(W^{n+1}(t))$.