WRITTEN EXAM – STOCHASTIC PROCESSES, 6.4.2017.

1. Suppose that 2 batteries are randomly chosen without replacement from the following group of 12 batteries: 3 new, 4 used (working) and 5 defective. Let X denote the number of new batteries chosen and let Y denote the number of used batteries chosen. Find $E(X)$ and $E(X|Y)$.

2. Let W be an exponential random variable with probability density function

$$\varphi_W(w) = \begin{cases} e^{-w}, & w \geq 0 \\ 0, & otherwise \end{cases}.$$

(a) Find the cumulative distribution function F_{X_t} of the time delayed ramp process $X_t = t - W$.

(b) Find the autocovariance function of a process X_t.

3. The diffusion of electrons and holes across a potential barrier in an electronic devise is modeled as follows. There are m black balls (electrons) in urn A and m white balls (holes) in urn B. We perform independent trials, in each of which a ball is selected at random from each urn and the selected ball from urn A is placed in urn B, while that from urn B is placed in A. Consider the Markov chain representing the number of black balls in urn A immediately after the $n-$th trial.

(a) Describe the one-step transition probabilities of the process.

(b) Suppose $m = 2$. Compute the long-run fraction of time when urn A does not contain a black ball.

4. Events occur according to a nonhomogeneous Poisson process whose mean value function is given by

$$m(t) = t^2 + 2t, \quad t \geq 0.$$

Find the intensity function of this process. What is the probability that n events occur between times $t = 4$ and $t = 5$?

5. Show that for any $T > 0$, $V(t) = W(t + T) - W(T)$ is a standard Brownian motion if $W(t)$ is.