WRITTEN EXAM – STOCHASTIC PROCESSES, 1.2.2017.

1. Consider the following joint probability density function

$$\varphi_{X,Y}(x,y) = \begin{cases} c\, e^{-(2x+3y)}, & x \geq 0, y \geq 0, \\ 0, & otherwise. \end{cases}$$

Determine the value of c and then determine the marginal probability density function of X and $P\{X > 1/2 \text{ and } Y > 1/3\}$.

2. A prisoner is trapped in a cell containing three doors. The first door leads to a tunnel that returns him to his cell after two days of travel. The second leads to a tunnel that returns him to his cell after three days of travel. The third door leads immediately to freedom.

(a) Assuming that the prisoner will always select doors 1, 2, and 3 with probabilities 0.5, 0.3, 0.2 respectively, what is the expected number of days until he reaches freedom?

(b) Assuming that the prisoner is always equally likely to choose among those doors that he has not used, what is the expected number of days until he reaches freedom? (In this version, for instance, if the prisoner initially tries door 1, then when he returns to the cell, he will now select only door 2 or 3.)

3. Ana is doing data analysis and she is receiving data through a communication system. But, she is aware that digit entered might be changed in the process and based of the prediction analysis she knows which digit is unchanged/changed. Previously, she received a transition probability matrix P for a three state Markov chain

$$P = \begin{bmatrix} a & b & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}.$$

Ana knows that, in the long run, 25% of the time system spends in the state 2. Find constants a and b, if possible.

4. A geiger counter is a device to count the radioactive particles emitted by a source. Suppose the particles arrive at the counter according to a Poisson process with rate $\lambda = 1000$ per second. The counter fails to count a particle with probability 0.1, independently of everything else. Suppose the counter registers four particles in 0.01 seconds. What is the probability that at least six particles have arrived at the counter during this time period?

5. Show that $e^{W_t} e^{-t/2}$ is a martingale if W_t is standard Brownian motion.

Help: $\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$.