Stochastic processes

Problem 1. Let

$$X(t) = at + X_0, \quad a \in \mathbb{R}, \quad t \in (-\infty, \infty),$$

where X_0 is a random variable with distribution function $F_{X_0}(x)$. Find the distribution function of the first and second order of the stochastic process X_t.

Problem 2. Let

$$X_t = a + tX_0, \quad t \in (-\infty, \infty), \quad a \in \mathbb{R},$$

where X_0 is a random variable with distribution function $F_{X_0}(x)$. Find the distribution function of the first order of stochastic process X_t.

Problem 3. Let X_t be the stochastic process given by

$$X_t = U + tV, \quad t \in \mathbb{R}$$

where U and V are two independent random variables. Calculate

- mean
- autocovariance function and
- variance

of X_t.

Problem 4. Find the mean, autocovariance function and variance of a stochastic process

$$X_t = \cos(\lambda t + U), \quad \lambda = const, \quad U : \mathcal{U}(0, 2\pi).$$

Problem 5. Let X and Y be two independent random variables, where

$$\varphi_X(x) = \begin{cases} \frac{4}{3} - x^2, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$$

and $Y : \mathcal{U}(0, \pi)$. Find a mean, autocovariance function, and variance of a stochastic process

$$U_t = X \cos(t - Y), \quad t \in \mathbb{R}$$

Problem 6. Let U and V be two independent random variables and let

$$U : \mathcal{U}(-2, 2), \quad V : \mathcal{N}(0, 1).$$

Find a mean, autocovariance function and variance of a stochastic process

$$X_t = V \cdot 2^t + U \cdot t^2, \quad t \in \mathbb{R}.$$