Introduction to Probability Theory

Problem 1. Bev can either take a course in computers or in chemistry. If Bev takes the computer course, then she will receive an A grade with probability $\frac{1}{2}$. If she takes the chemistry course then she will receive an A grade with probability $\frac{1}{3}$. Bev decides to base her decision on the flip of a fair coin. What is the probability that Bev will get an A in chemistry?

Problem 2. Integrated circuits undergo two tests. A mechanical test determines whether pins have the correct spacing, and an electrical test checks the relationship of outputs to inputs. We assume that electrical failures and mechanical failures occur independently. Our information about circuit production tells us that mechanical failures occur with probability 0.05 and electrical failures occur with probability 0.2. What is the probability model of an experiment that consists of testing an integrated circuit and observing the results of the mechanical and electrical tests?

Problem 3. Consider an experiment involving two successive rolls of a 4-sided die in which all 16 possible outcomes are equally likely and have probability 1/16.

(a) Are the events

$$A_i = \{\text{1st roll results in } i\}, \quad B_j = \{\text{2st roll results in } j\}$$

independent?

(b) Are the events

$$A = \{\text{1st roll is a 1}\}, \quad B = \{\text{sum of the two rolls is a 5}\}$$

independent?

(c) Are the events

$$A = \{\text{maximal of the two rolls is 2}\}, \quad B = \{\text{minimum of the two rolls is 2}\}$$

independent?

Problem 4. A class consisting of 4 graduate and 12 undergraduate students is randomly divided into 4 groups of 4. What is the probability that each group includes a graduate student? We interpret "randomly" to mean that given the assigment of some students to certain slots, any of the remaining students is equally likely to be assigned to any of the remaining slots.

Problem 5. Suppose traffic engineers have coordinated the timing of two traffic lights to encourage a run of green lights. In particular, the timing was designed so that with probability 0.8 a driver will find the second light to have the same color as the first. Assuming the first light is equally likely to be red or green, what is the probability $P[G_2]$ that the second light is green? Also, what is $P[W]$, the probability that you wait for at least one light? Lastly, what is $P[G_1|R_2]$, the conditional probability of a green first light given a red second light?

Problem 6. A laboratory blood test is 95 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1 percent of the healthy persons tested (If a healthy person is tested, then, with probability 0.01, the test result will imply he has the disease). If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that his test result is positive?

Problem 7. Stores A, B and C have 50, 75 and 100 employees, and respectively 50, 60 and 70 percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns and this is a woman. What is the probability that she works in store C?

Problem 8. The dice game craps is played as follows. The player throws two dice and if the sum is seven or eleven, then she wins. If the sum is two, three or twelve, then she loses. If the sum is anything else, then she continues throwing until she either throws that number again (in which case she wins) or she throws a seven (in which case she loses). Calculate the probability that the player wins.

Problem 9. Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

(a) What is the probability that a product attains a good review?

(b) If a new design attains a good review, what is the probability that it will be a highly successful product?