# Poisson process

1. The count of students dropping the course "Probability and Stochastic Processes" is known to be a Poisson process of rate 0.1 drops per day. Starting with day 0, the first day of the semester, let $D_t$ denote the number of students that have dropped after $t$ days. What is $P\{D_t = k\}$?

2. The arrivals of new telephone calls at a telephone switching office is a Poisson process $X_t$ with an arrival rate of $\lambda = 4$ calls per second. An experiment consists of monitoring the switching office and recording $X_t$ over a 10-second interval.

   **(a)** What is $P\{X_1 = 0\}$, the probability of no phone calls in the first second of observation?

   **(b)** What is $P\{X_1 = 4\}$, the probability of exactly four calls arriving in the first second of observation?

   **(c)** What is $P\{X_2 = 2\}$, the probability of exactly two calls arriving in the first two seconds?

3. The stochastic process $\{X_t, t \geq 0\}$ is defined by

$$X_t = N_{t+1} - N_1 \text{ for } t \geq 0$$

   where $\{N_t, t \geq 0\}$ is a Poisson process with rate $\lambda > 0$. Find a $K_X(s,t)$, for $0 \leq s \leq t$.

4. Let $X(t)$ be a Poisson process with parameter $\lambda$. Find

   $(a)$ $E[X^2(t)]$

   $(b)$ $E[(X(t) - X(s))^2]$, for $t > s$.

5. The number of failures $N_t$, which occur in a computer network over the time interval $[0,t)$, can be described by a homogeneous Poisson process $\{N_t, t \geq 0\}$. On an average, there is a failure after every 4 hours, i.e. the intensity of the process is equal to $\lambda = 0.25$.

   **(a)** What is the probability of at most 1 failure in $[0,8)$, at least 2 failures in $[8,16)$ and at most 1 failure in $[16,24)$ (time unit: hour)?

   **(b)** What is the probability that the third occurs after 8 hours?

6. A sequence of queries are made to a database system. The response time of the system, $T$ seconds, is an exponential random variable with mean 8. As soon as the system responds to a query, the next query is made. Assuming the first query is made at time zero, let $N_t$ denote the number of queries made by time $t$.

   **(a)** What is $P\{T \geq 4\}$, the probability that a single query will last at least four seconds?

   **(b)** If the database user has been waiting five seconds for a response, what is $P\{T \geq 13 | T \geq 5\}$, the probability that it will last at least eight more seconds?

   **(c)** What is the PMF of $N_t$?

7. Suppose that people immigrate into a territory at a Poisson rate $\lambda = 1$ per day.
   **(a)** What is the expected time until the tenth immigrant arrives?
   **(b)** What is the probability that the elapsed time between the tenth and the eleventh arrival exceeds two days?
8. Customers arrive at a casino as a Poisson process of rate 100 customers per hour. Upon arriving, each customer must flip a coin, and only those customers who flip heads actually enter the casino. Let $X_t$ denote the process of customers entering the casino. Find the PMF of $N$, the number of customers who arrive between 5 p.m. and 7 p.m..
9. Consider an elevator that starts in the basement and travels upward. Let $N_i$ denote the number of people that get in the elevator at floor $i$. Assume the $N_i$ are independent and that $N_i$ is Poisson random variable with mean $\lambda_i$. Each person entering at $i$ will, independent of everything else, get off at $j$ with probability $p_{ij}$, $\sum_{j>i} p_{ij} = 1$. Let $O_j$ denote the number of people getting off the elevator at floor $j$.
   **(a)** Compute $E(O_j)$.
   **(b)** What is the distribution of $O_j$?
   **(c)** What is the joint distribution of $O_j$ and $O_k$?
10. For a nonhomogeneous Poisson process $N_t$ the intensity function is given by
$$\lambda(t) = \begin{cases} 5, & \text{if t is in } (1,2], (3,4], \ldots \\ 3, & \text{if t is in } (0,1], (2,3], \ldots \end{cases}.$$
    Find the probability that the number of observed occurrences in the time period $(1.25, 3]$ is more than two.
11. A store opens at 8 a.m. From 8 until 10 customers arrive at a Poisson rate of four per hour. Between 10 and 12 they arrive at a Poisson rate of eight per hour. From 12 to 2 p.m. the arrival rate increases steadily from eight per hour at 12 to ten per hour at 2 p.m. and from 2 to 5 p.m. the arrival rate drops steadily from ten per hour at 2 to four per hour at 5 p.m.. Determine the probability distribution of the number of customers that enter the store on a given day.
12. Suppose that health claims are filed with a health insurer at the Poisson rate $\lambda = 20$ per day, and that the independent severities of each claim are Exponential random variables with mean $\theta = 500$. Find the expected value and variance of an aggregate of claims during the first 10 days. Estimate the probability that the aggregate claims during the first 10 days exceed 120 000.
13. A geiger counter is a device to count the radioactive particles emitted by a source. Suppose the particles arrive at the counter according to a Poisson process with rate $\lambda = 1000$ per second. The counter fails to count a particle with probability 0.1, independent of everything else. Suppose the counter registers four particles in 0.01 seconds. What is the probability that at least six particles have actually arrived at the counter during this time period?

14. Events occur according to a nonhomogeneous Poisson process whose mean value function is given by

$$m(t) = t^2 + 2t, \quad t \geq 0.$$

Find the intensity function of this process. What is the probability that $n$ events occur between times $t = 4$ and $t = 5$?

15. In good years, storms occur according to a Poisson process with rate 3 per unit time, while in other years they occur according to a Poisson process with rate 5 per unit time. Suppose next year will be a good year with probability 0.3. Let $N_t$ denote the number of storms during the first $t$ time units of next year.

   ($a$) Find the probability that in the first $t$ time units of next year have occurred $n$ storms.

   ($b$) Is $\{N_t, \, t \geq 0\}$ a Poisson process?

   ($c$) Does $\{N_t, \, t \geq 0\}$ have stationary increments? Explain.

   ($d$) If next year starts off with three storms by time $t = 1$, what is the probability it is a good year?

16. The number of trains from Novi Sad to Belgrade which are late more than one hour is described by a Poisson process with a rate one per month, while number of trains from Novi Sad to Subotica which are late more than one hour is an independent Poisson process with a rate 2 per month.

   ($a$) What is the probability that at least 3 trains from Novi Sad to Belgrade are late more then one hour during one month?

   ($b$) What is the probability that 3 trains from Novi Sad to Belgrade and 2 trains from Novi Sad to Subotica are late more that 1 hour during 1 month?

17. All the employees in one company use the same printer. Suppose that the times between two consecutive print requests are independent, identically distributed exponential random variables with parameter 10 per hour. The printer needs exactly 6 seconds to print one paper.

   ($a$) What is the probability that exactly 20 print requests will arrive between $8:30$ i $10:30$h?

   ($b$) What is the probability that a print request will arrive while the previous document containing 6 papers is not yet printed?

18. The number of babies born is Serbia during $t$ days is a Poisson process. On an average, a baby is born after every 60 seconds. If it is known that in one of 10 cases baby is not discharged from the hospital after 5 days, find the probability that in two days period, the number of babies which are not discharged from the hospital after 5 days is at most 2.