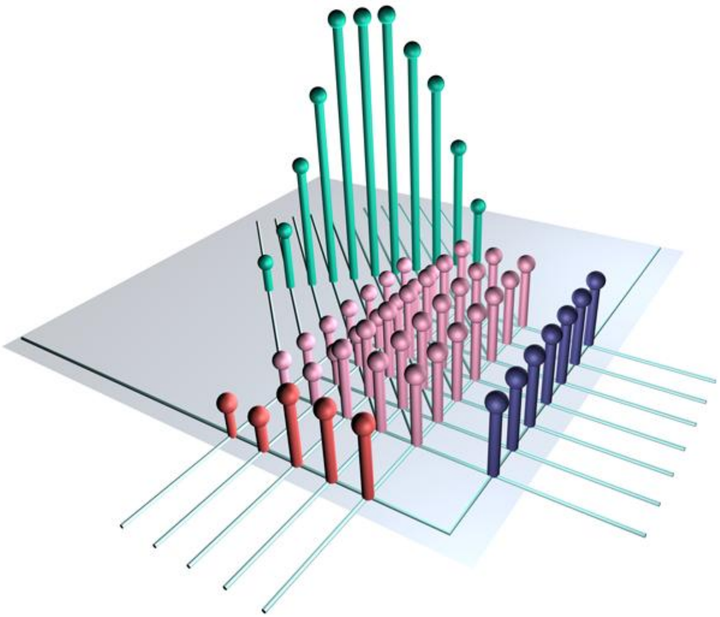
# INTRODUCTION TO DIGITAL SIGNAL PROCESSING



Lecturer:

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Faculty of Technical Sciences
University of Novi Sad

# DISCRETE-TIME
# SIGNALS AND SYSTEMS

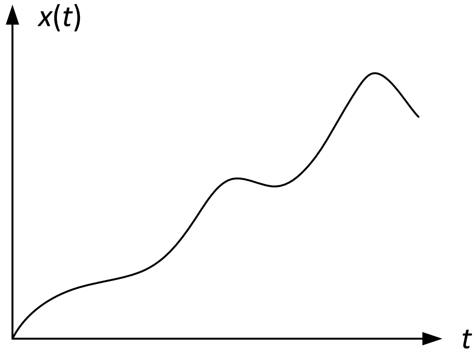
# Signal

- A measurable physical quantity used to transmit *messages*, i.e. *information*
- From a mathematical standpoint, a signal is no different than a *function* (a *mapping*)

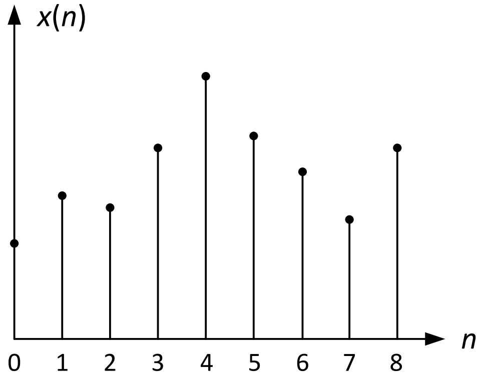
# Signal

- Independent variable:
  - Time, space coordinate or something else
  - One or more independent variables
  - Continuous or discrete

$x(t)$: $\mathbf{R} \rightarrow \mathbf{R}$




$x(n)$: $\mathbf{Z} \rightarrow \mathbf{R}$




- Dependent variable:
  - Set of real or complex numbers
  - Continuous or discrete

# Why digital?

- Digital signals are not so easily damaged
  - any impairment that is small enough can be completely removed from the signal
- Digital data lends itself to new concepts:
  - error detection and correction
  - encryption
  - compression
  - time domain multiplex
  - digital signal processing

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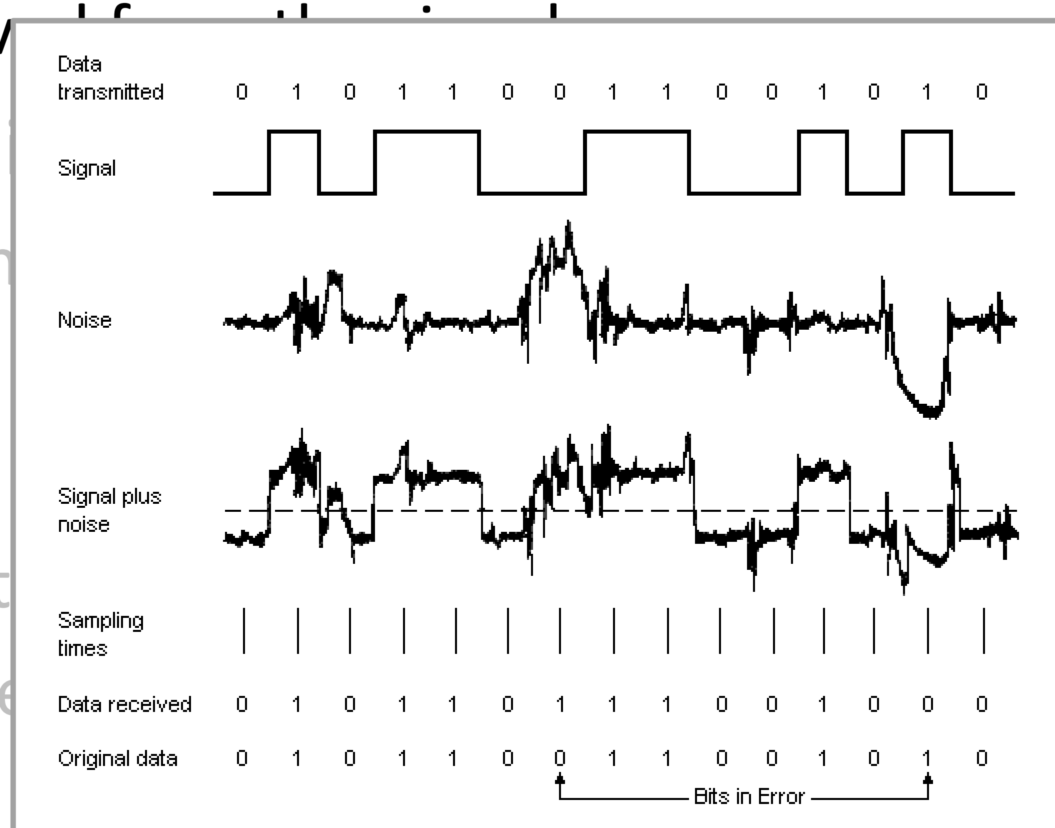

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> - *Checksum*
> - Cyclic redundancy check (CRC)
> - Codes based on Hamming distance
> - *Hash* functions
> - Turbo codes
>
> . . .

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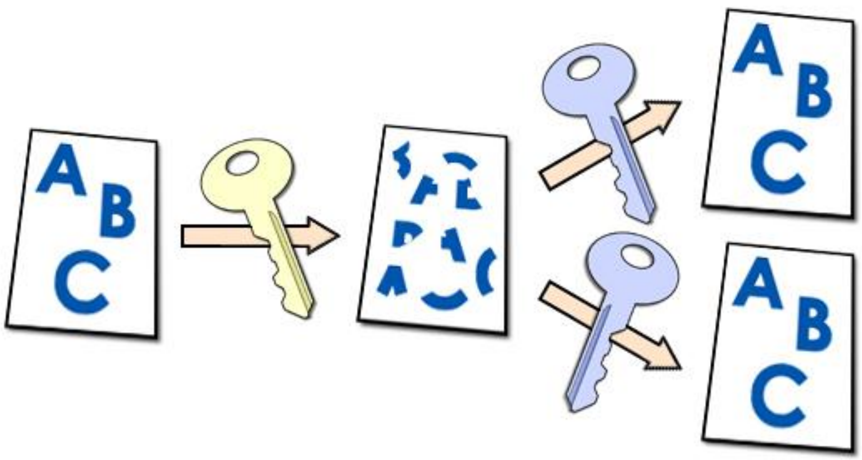
- Symmetric encryption
  - scrambling
- Asymmetric encryption
  - secret and public key



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- Compression
  - lossless (ZIP, RAR...)
  - lossy (JPG, MP3...)

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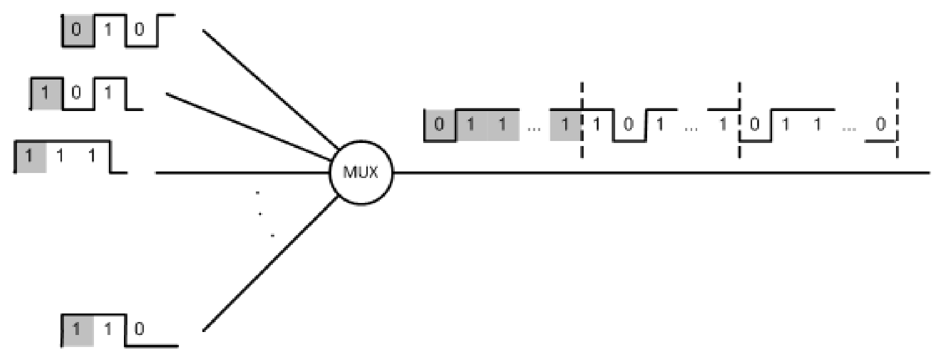


- Example: stereo WAV file

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- Flexibility
- Programmability
- Accuracy
- Stability
- Repeatability
- Small dimensions
- Price

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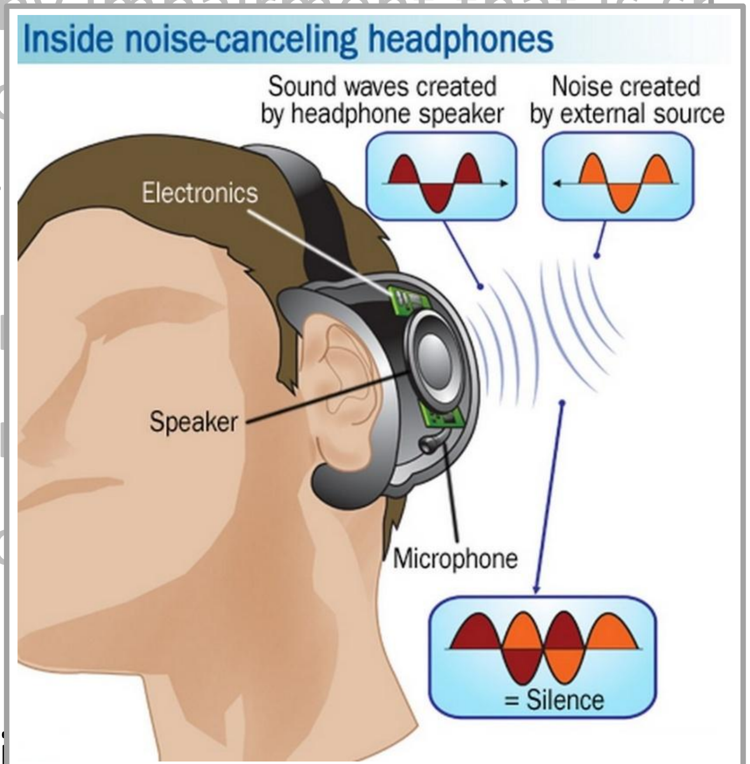


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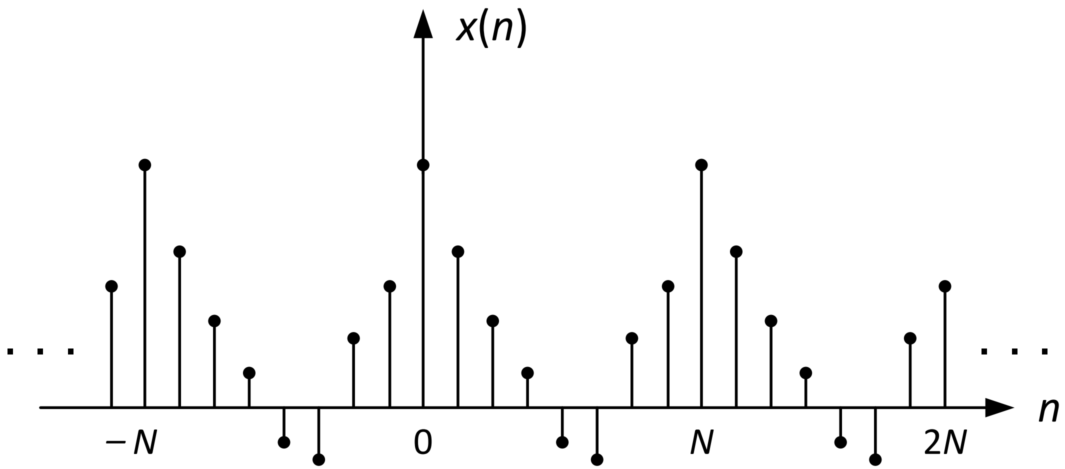
# Some applications of digital signal processing

- Telephony and communication systems
  - Speech compression
  - Channel coding
  - Speech and data processing
  - Echo cancellation
  - Noise reduction
  - Encryption
  - Generation and detection of DTMF signals
  - Power consumption management
- Personal computers
  - Sound and image processing
  - Multimedia
  - Modem communications
  - Internet telephony and video
- Car industry
  - Engine control and monitoring
  - Parking assistance
  - Autonomous navigation
  - Active safety
- Security systems
  - User authentication
  - Video surveillance
- Speech technology
  - Text-to-speech synthesis
  - Automatic speech recognition
  - Automatic speaker recognition
  - Interactive human-machine dialogue
- Medical electronics
  - Intensive care monitoring
  - EKG and EEG analysis
  - Medical image processing
- Digital audio
  - CD and DVD
  - Sound compression
  - Sound reproduction standards
  - Digital audio-effects
  - Noise reduction in audio
  - Electronic music
- Digital television
  - Sound and image processing
  - Video on demand
  - TV signal encryption

# Properties of discrete-time signals

## Periodicity

$$\exists N \in \mathbf{N}, x(n) = x(n+N)$$



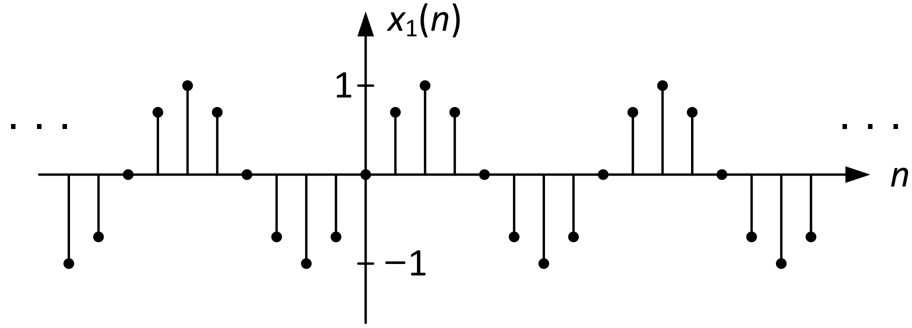

The smallest such $N$ is the *fundamental period* of the signal.

If there is no such $N$, the signal is *aperiodic*.

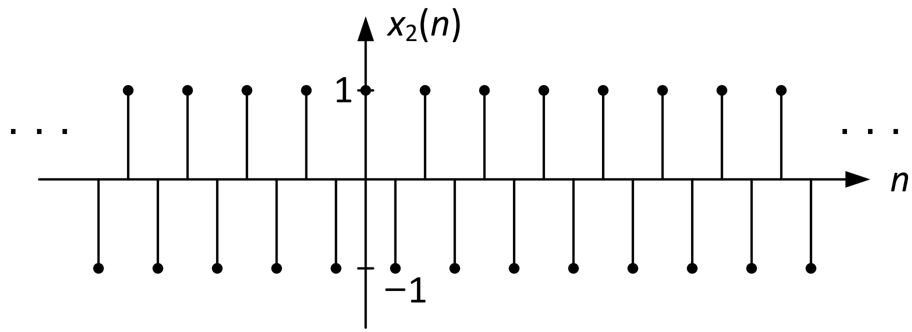
# Properties of discrete-time signals

## Sinusoid signal

$$\exists A, \omega, \varphi \in \mathbf{R}, x(n) = A\sin(\omega n + \varphi)$$




$$x_1(n) = \sin\frac{n\pi}{4}$$

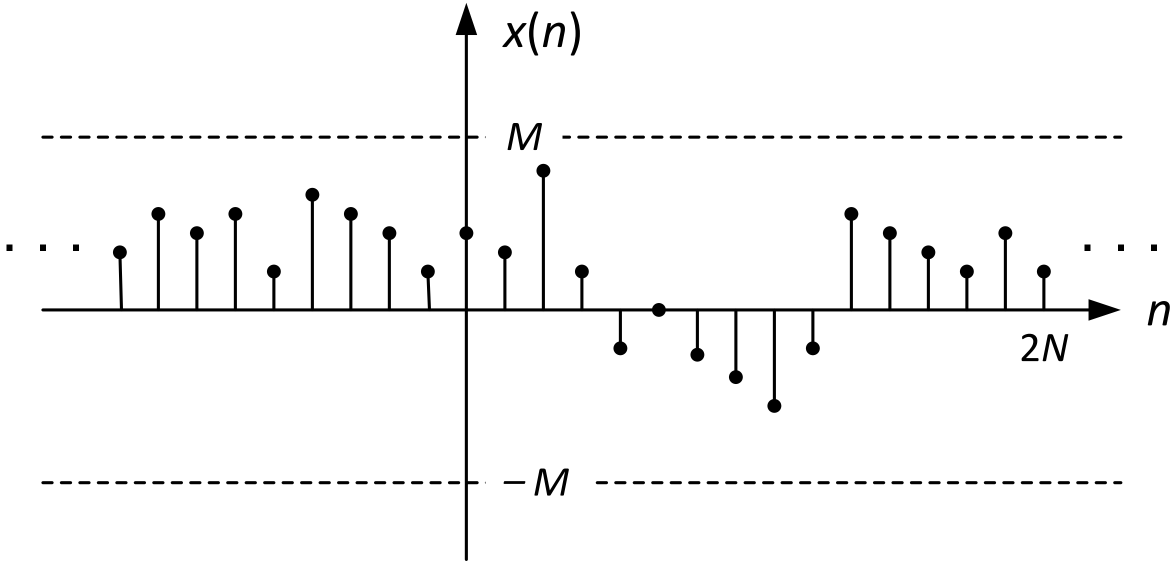



$$x_2(n) = \cos n\pi$$

# Properties of discrete-time signals

**Bounded signal**

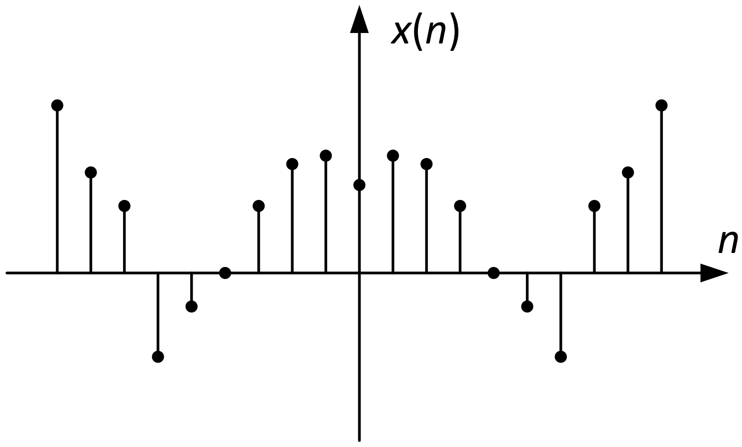
$$\exists M \in \mathbf{R}, |x(n)| < M$$



# Properties of discrete-time signals

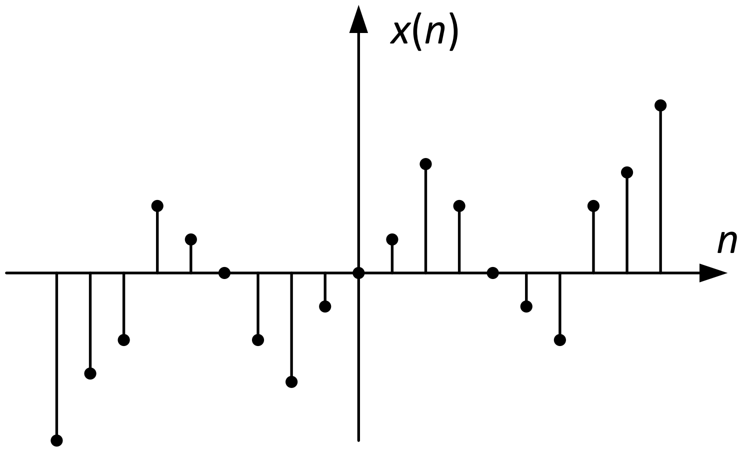
**Even signal**

$$\forall n, x(-n) = x(n)$$




**Odd signal**

$$\forall n, x(-n) = -x(n)$$

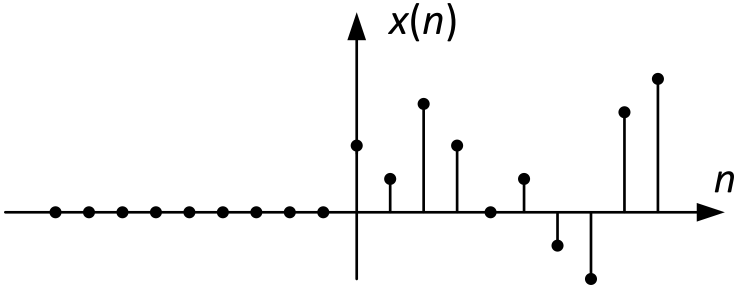



Each discrete-time signal $x(n)$ can be *uniquely represented* as a sum of one even and one odd signal

# Properties of discrete-time signals

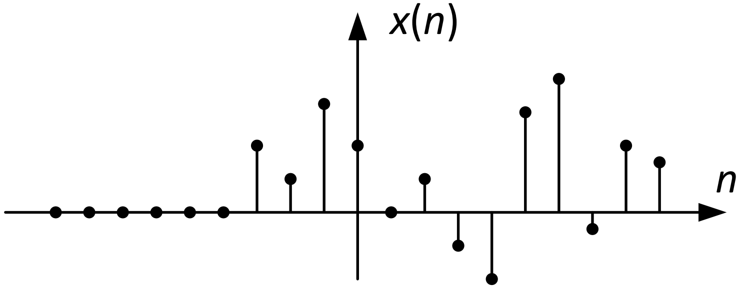
## Causality

$$\forall n < 0, x(n) = 0$$




## Wide-sende causality

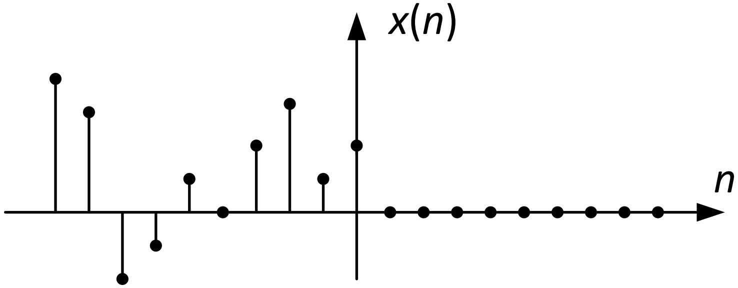
$$\exists N_1 \in \mathbf{Z},\ \forall n < N_1, x(n) = 0$$



# Properties of discrete-time signals

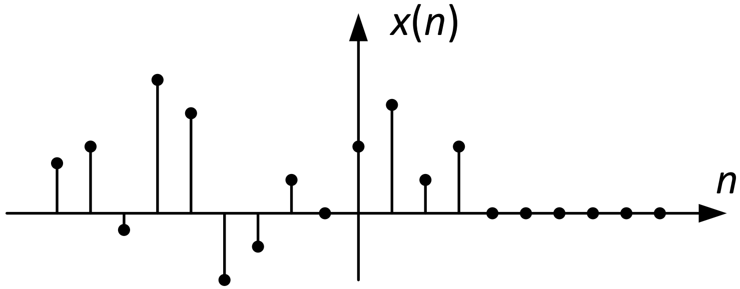
## Anticausality

$$\forall n > 0, x(n) = 0$$




## Wide-sense anticausality

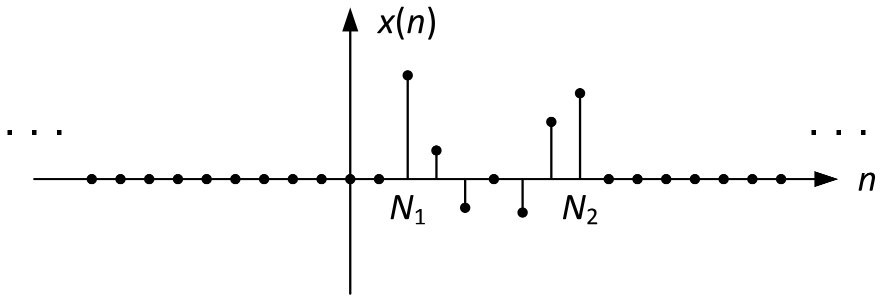
$$\exists N_1 \in \mathbf{Z},\ \forall n > N_1,\ x(n) = 0$$



# Duration of discrete-time signals

## Finite

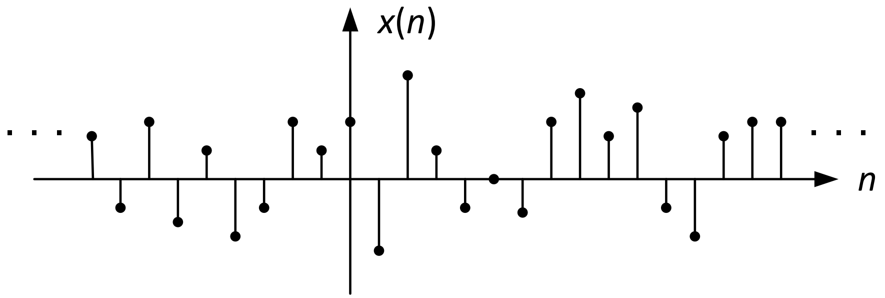
$$\exists N_1, N_2 \in \mathbf{Z}$$
$$n < N_1 \vee n > N_2, x(n) = 0$$




If $x(N_1) \neq 0$ i $x(N_2) \neq 0$, the duration of the signal is $N_2$-$N_1$+1.

## Infinite

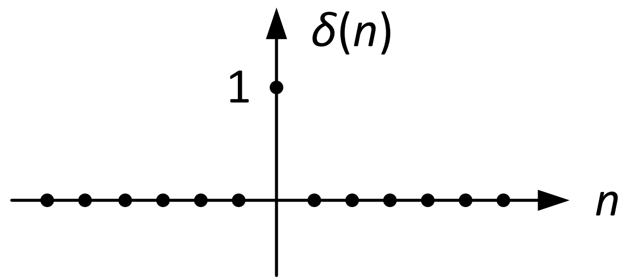
$$\nexists N_1, N_2 \in \mathbf{Z}$$
$$n < N_1 \vee n > N_2, x(n) = 0$$



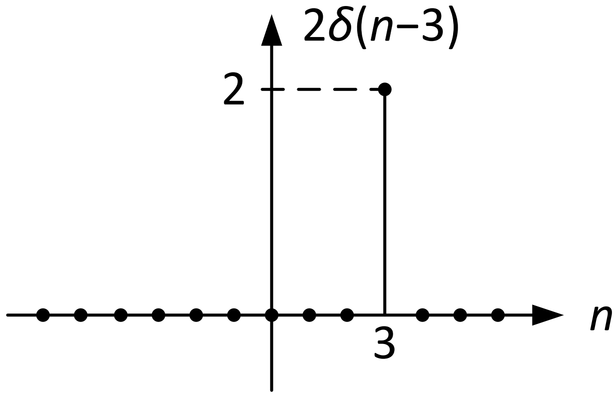
# Representation of discrete-time signals

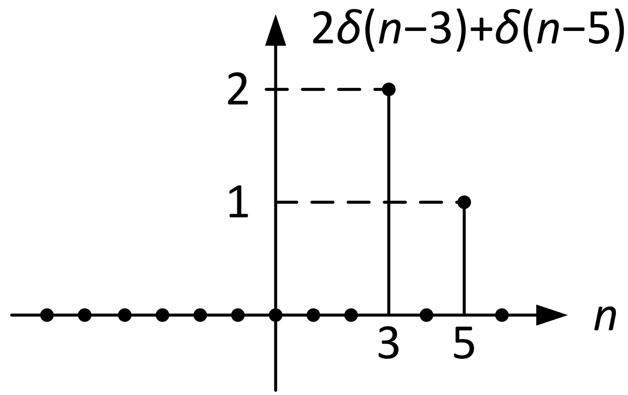
## Discrete-time δ-impulse

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$




Each discrete-time signal can be written as a linear combination of $\delta$-impulses shifted in time

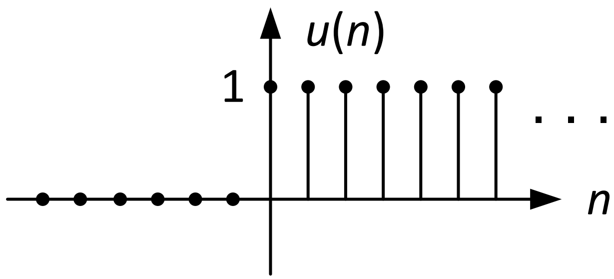


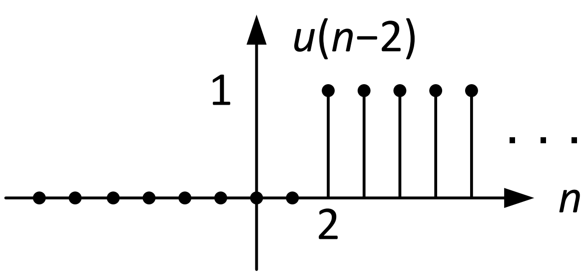
# Representation of discrete-time signals

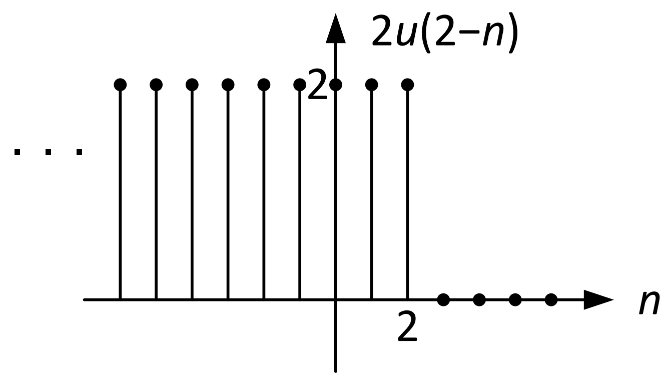
## Heaviside's impulse train

$$u(n)=\begin{cases}1, & n\geq 0\\ 0, & n<0\end{cases}$$




Some signals can be efficiently represented in terms of Heaviside's impulse train:



# Representation of discrete-time signals

Plot the following signals against time:

$$x_1(n) = u(n) - 2u(n-3)$$

$$x_2(n) = (n+3)u(n)$$

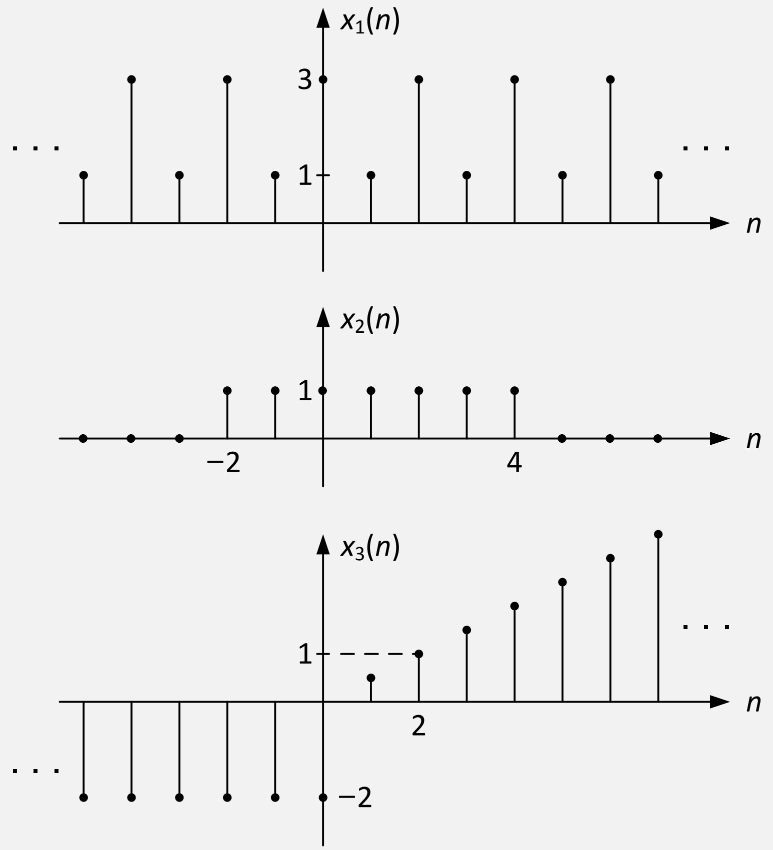
$$x_3(n) = \sin(n\pi / 2)u(-n)$$

$$x_4(n) = \sum_{k=0}^{\infty} \delta(n-3k)$$

$$x_5(n) = 2^n u(3-n)$$

# Representation of discrete-time signals

Analytically describe the following signals:



# Convolution

## Linear convolution

$$l(n) = a(n) * b(n) = \sum_{k=-\infty}^{\infty} a(k)b(n-k)$$

- Does not always exist
- Sufficient condition for its existence is that either $a(n)$ or $b(n)$ should have finite duration
- If they are both of finite durations ($N_1$ and $N_2$ respectively), the duration of the signal $l(n)$ is $N_1+N_2-1$
- Commutative operation, the neutral element is $\delta(n)$

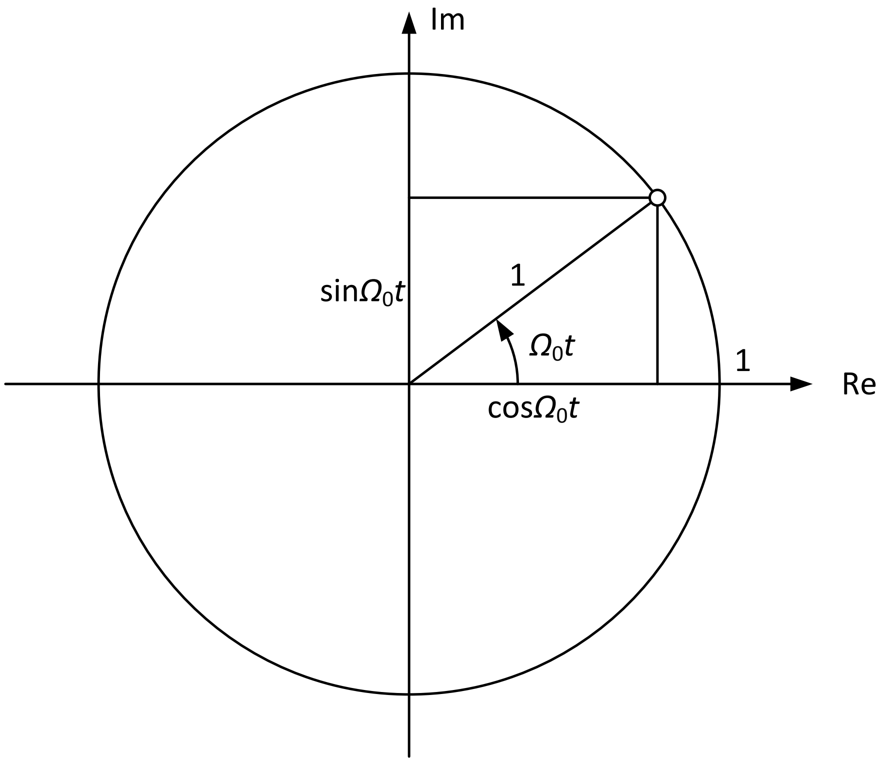
## Circular (cyclic) convolution

$$c(n) = a(n) \circledast b(n) = \sum_{k=0}^{N-1} a(k)b(n-k)$$

- Defined for periodic signals of periods equal to $N$
- The result is also a periodic signal whose period is $N$

# Properties of sinusoidal signals

- To begin with, let us analyze a *continuous-time* sinusoidal signal




$$s(t) = \cos\Omega_0 t + j\sin\Omega_0 t$$

$$s(t) = e^{j\Omega_0 t}$$

Fundamental frequency:

line: $f_0$ [Hz]

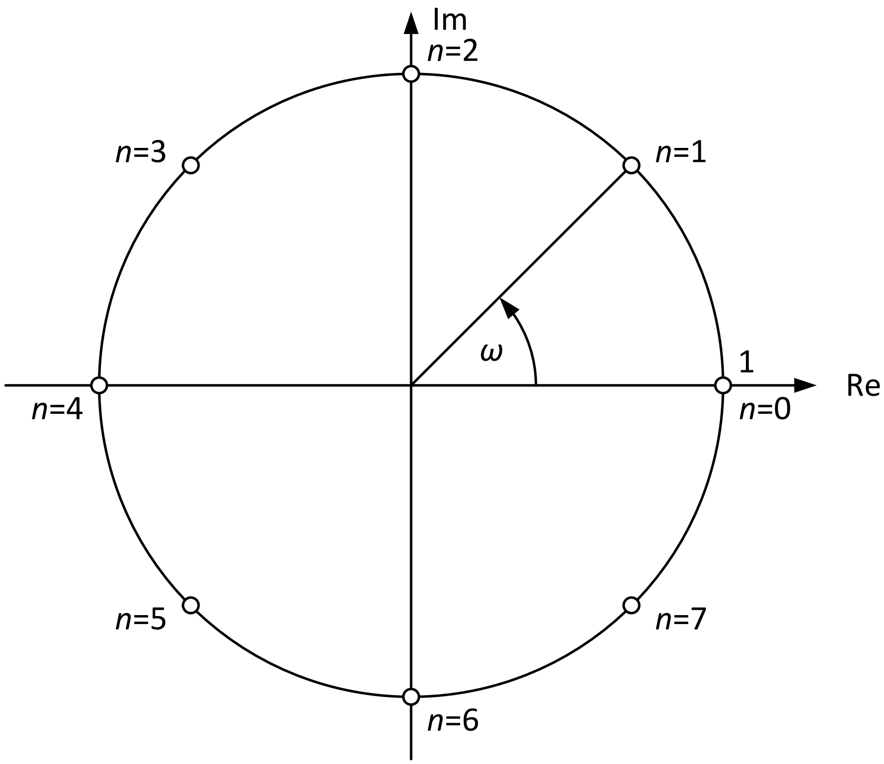
angular: $\Omega_0$ [rad/s]

$$\Omega_0 = 2\pi f_0$$

Where do we encounter the signal $s(t) = e^{j\Omega_0 t}$ in continuous-time signal processing?

# Properties of sinusoidal signals

- Now let us analyze the same signal in *discrete time*




Cycles per second: $f_0$ = 1 Hz

Observations per second: $f_s$ = 8 Hz

Time between observations: $T = 1/f_s$

$$s(nT) = e^{jn\Omega_0 T} = e^{j\omega_0 n} = s_d(n)$$

Number of cycles between two observations defines the *frequency of the discrete signal*

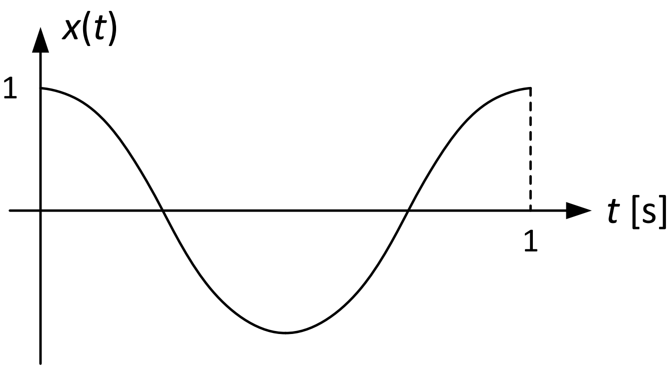
$\xi = fT = f/f_s$ [cycles per observation]

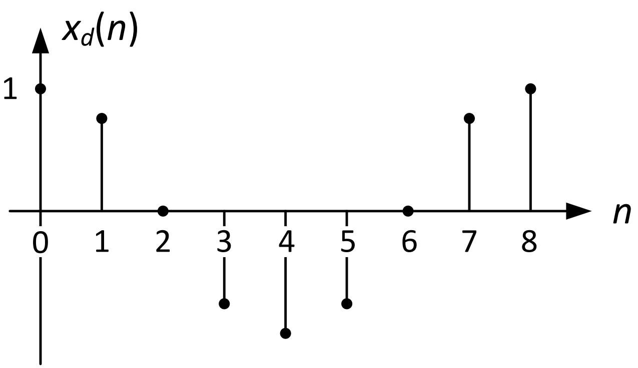
Phase angle between two observations defines the *angular frequency of the discrete signal*

$\omega = \Omega T = \Omega/f_s$ [radians per observation]

# Properties of sinusoidal signals

- This is, in fact, the *sampling* of a sinusoidal signal




Cycles per second: $f_0$ = 1 Hz

Observations per second: $f_s$ = 8 Hz

Time between observations: $T = 1/f_s$

$$x(t) = \cos \Omega_0 t$$

$$x_d(n) = x(nT) = \cos \Omega_0 nT = \cos \omega_0 n$$

$$\Omega_0 = 2\pi f_0 = 2\pi \text{ rad/s}$$

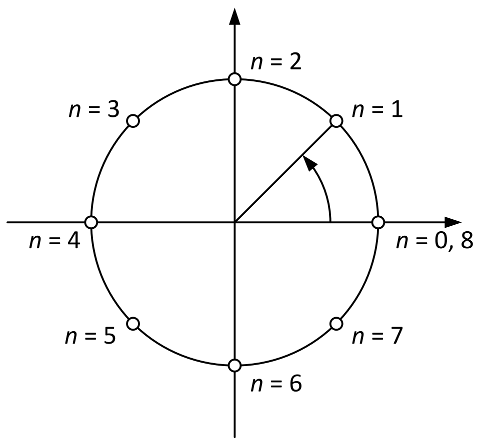
$$\omega_0 = \Omega_0 T = \frac{\Omega_0}{f_s} = \frac{\pi}{4} \qquad \xi_0 = \frac{f_0}{f_s} = \frac{1}{8}$$

# The notion of frequency

- The notion of frequency is different with continuous-time (analogue) signals and discrete-time signals

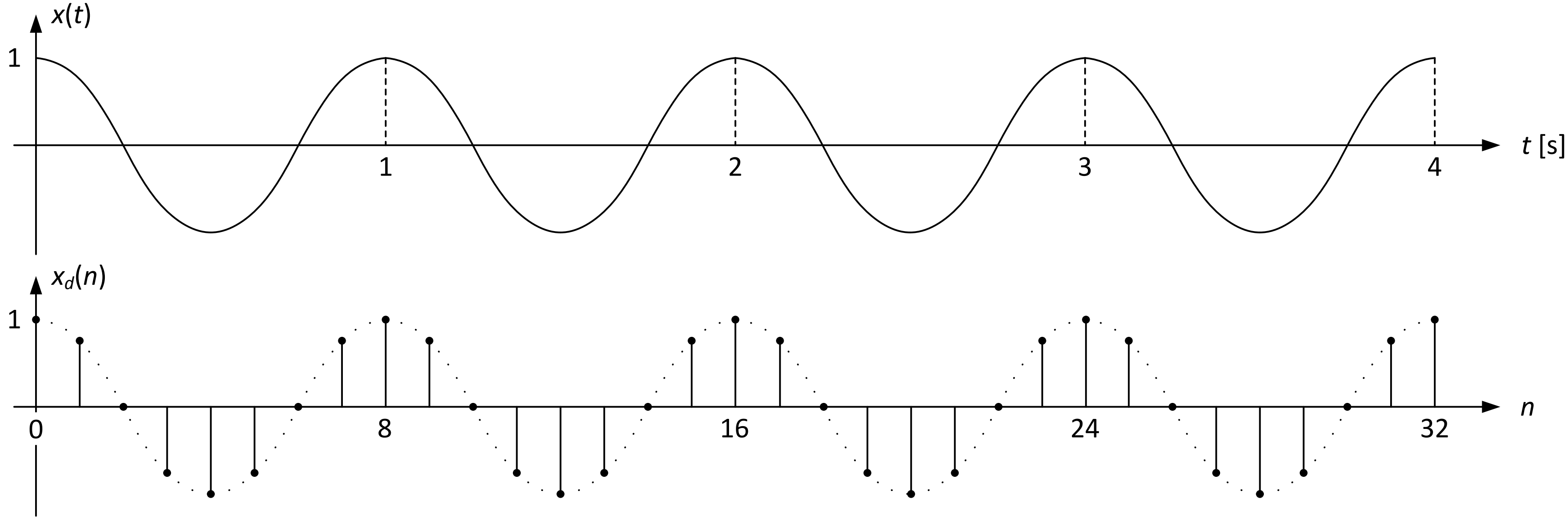
| | FREQUENCY | |
|---|---|---|
| | LINE | ANGULAR |
| CONTINUOUS-TIME SIGNALS (period *T*) | $f = \frac{1}{T}$ [Hz] | $\Omega = \frac{2\pi}{T} = 2\pi f$ [rad/s] |
| DISCRETE-TIME SIGNALS (period *N*) | $\xi = f T_s = \frac{f}{f_s}$ [ ] | $\omega = \Omega T_s = \frac{\Omega}{f_s} = 2\pi\xi$ [rad] |

# Examples of sampling




$$f_0 = 1\,\text{Hz} \qquad \Omega_0 = 2\pi\,\text{rad/s}$$

$$\xi_0 = \frac{1}{8} \qquad \omega_0 = \frac{\pi}{4}$$



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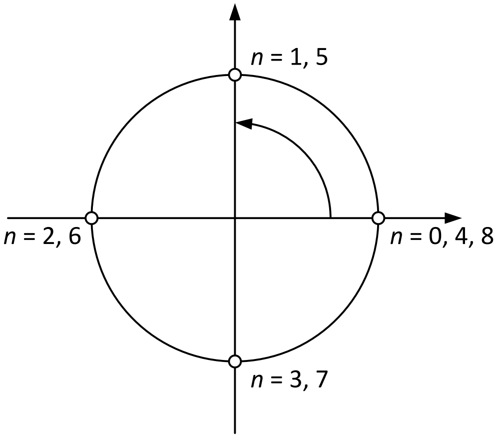


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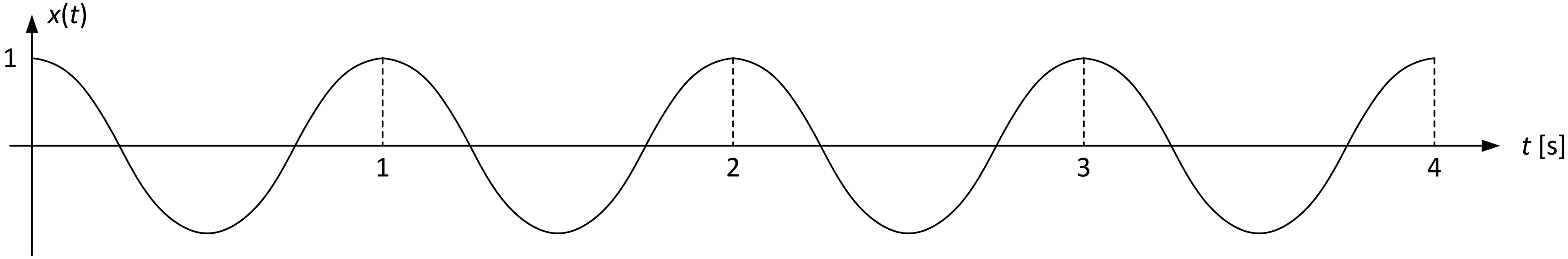


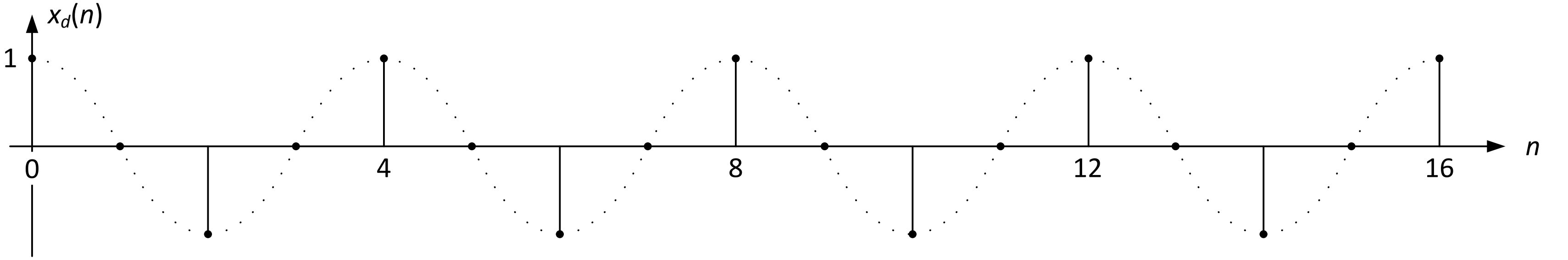

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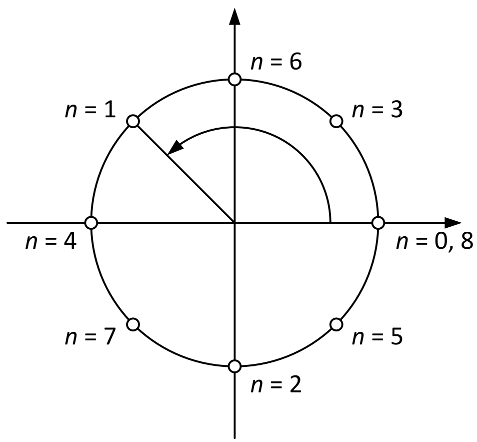
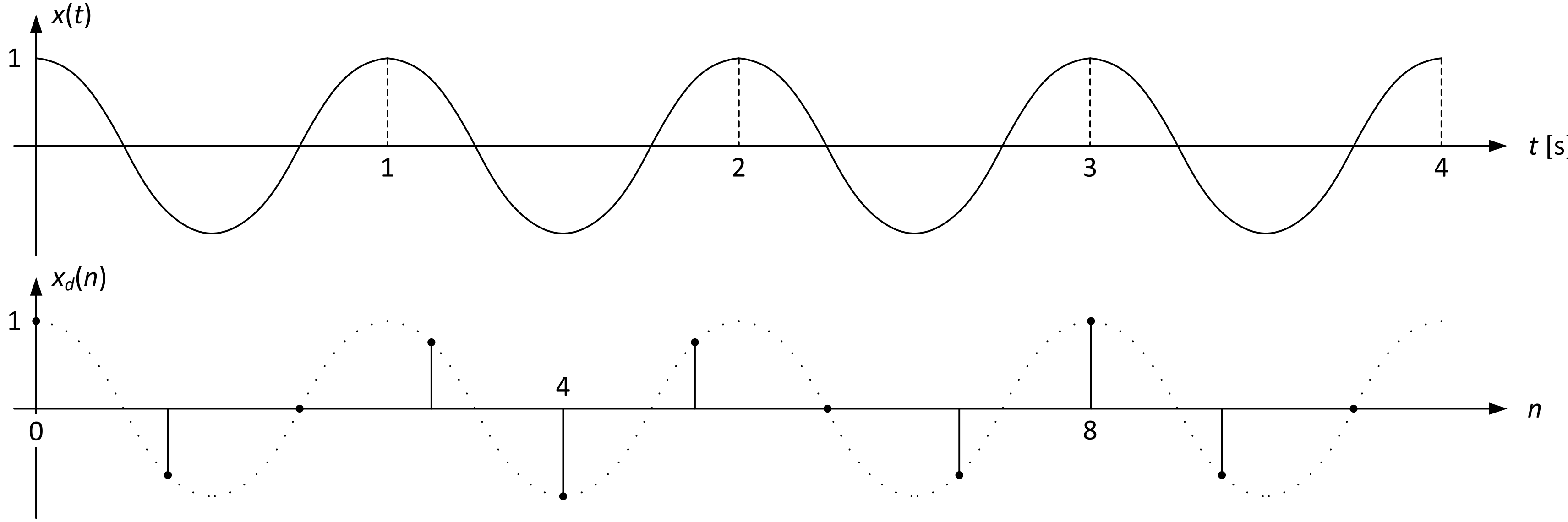



$$f_0 = 1\,\text{Hz} \qquad \Omega_0 = 2\pi\,\text{rad/s}$$

$$\xi_0 = \frac{3}{8} \qquad \omega_0 = \frac{3\pi}{4}$$



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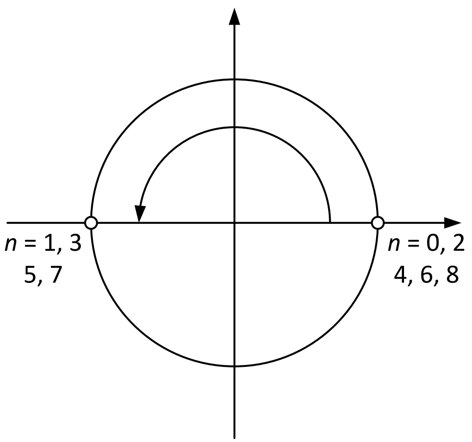


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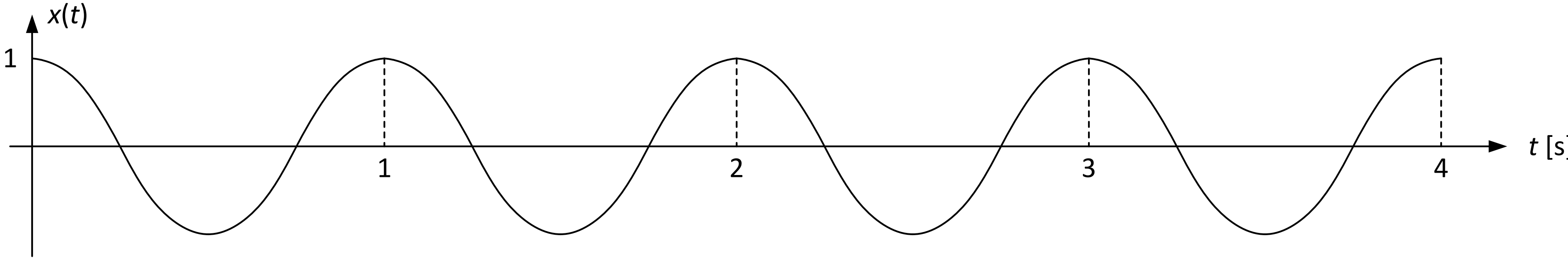
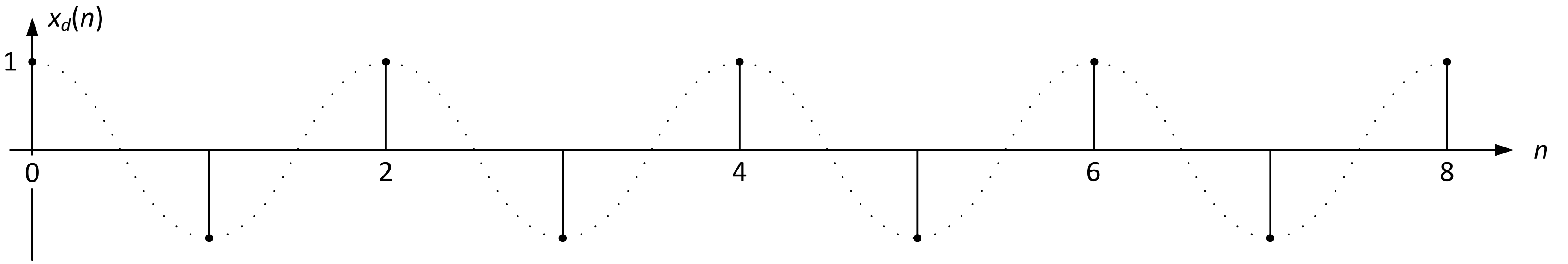
$$\xi_0 = \frac{1}{2} \qquad \omega_0 = \pi$$



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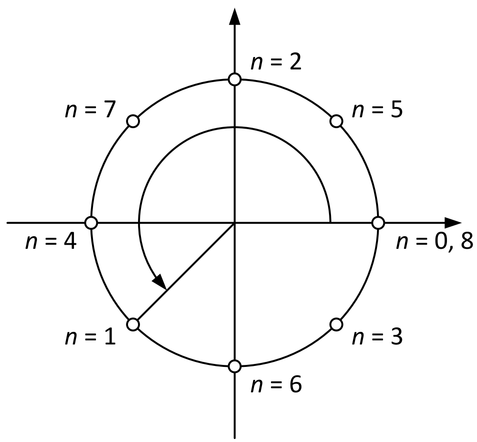


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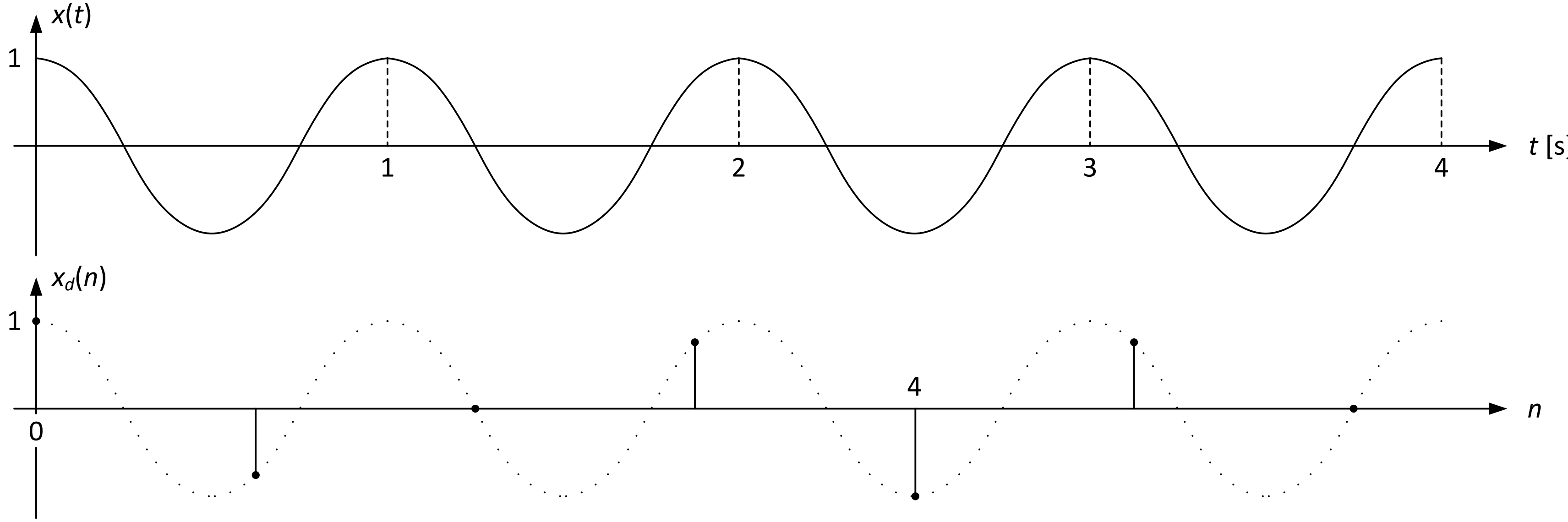

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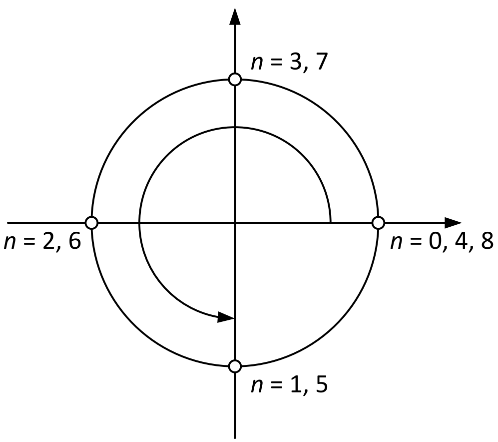


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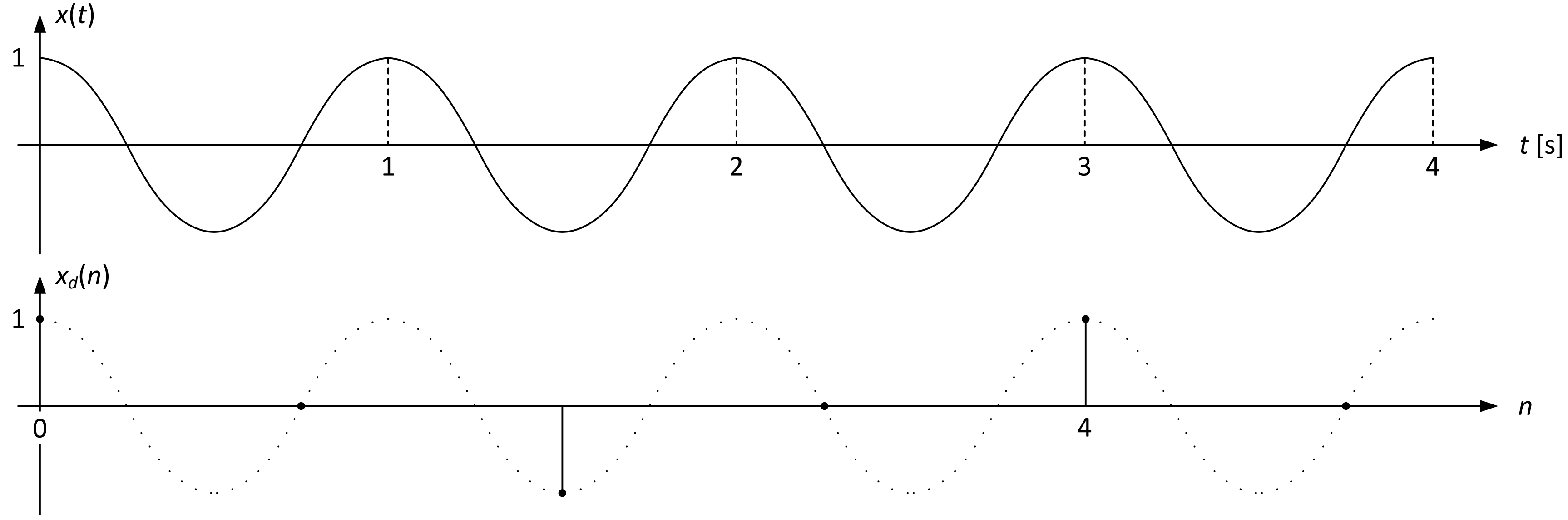

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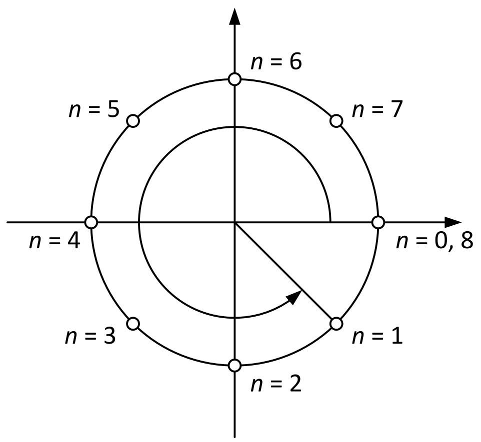
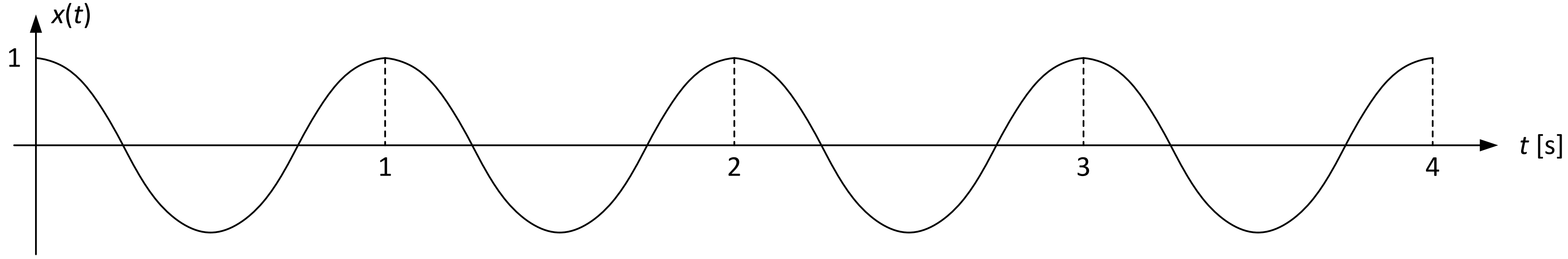
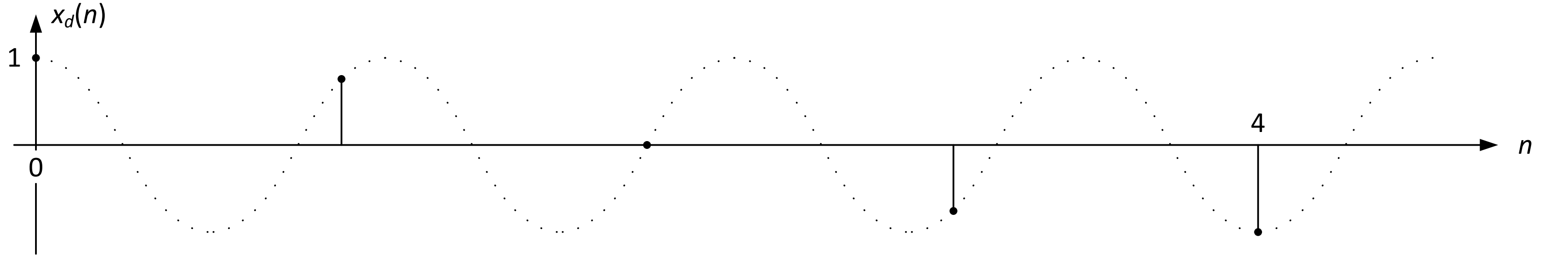


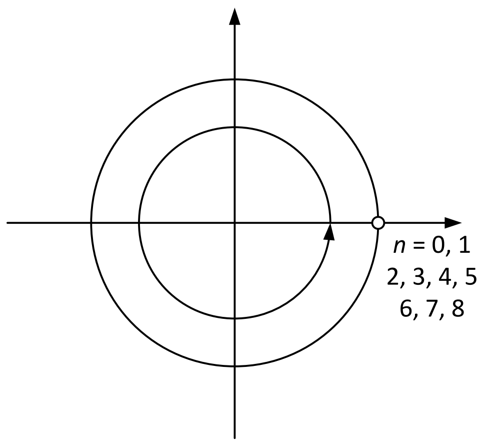

$$f_0 = 1\,\text{Hz} \qquad \Omega_0 = 2\pi\,\text{rad/s}$$

$$\xi_0 = \frac{7}{8} \qquad \omega_0 = \frac{7\pi}{4}$$



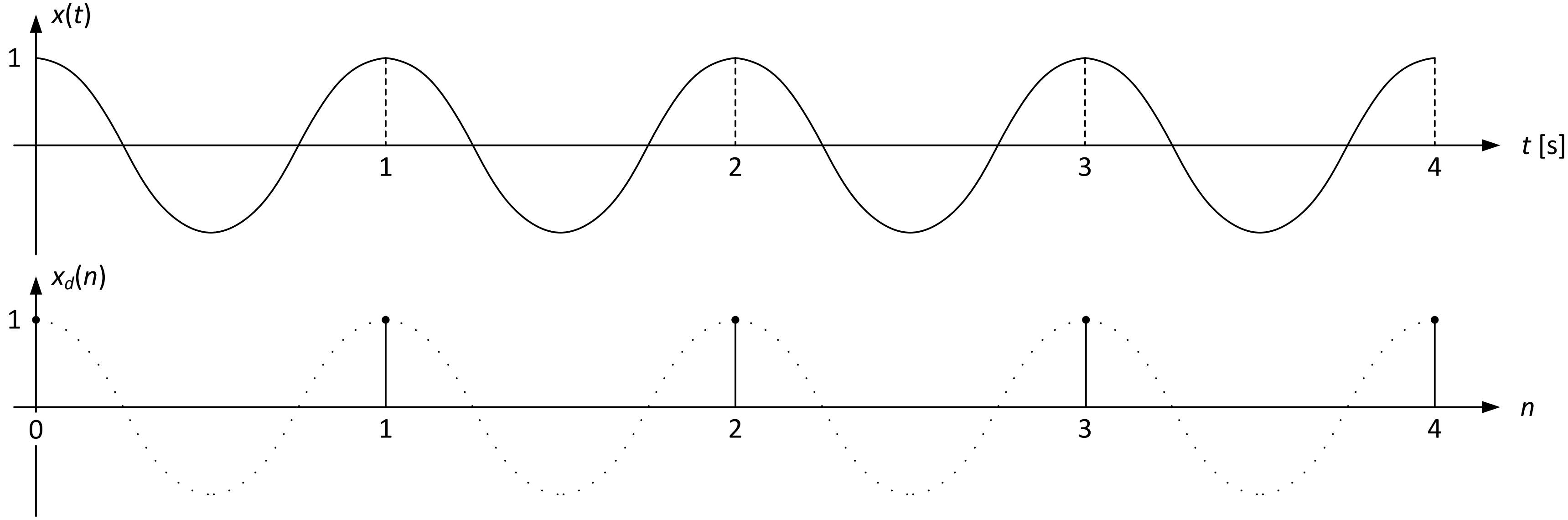



# Examples of sampling

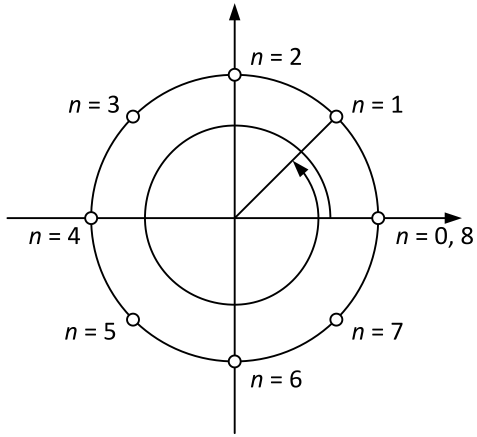



$$f_0 = 1\,\text{Hz} \quad \Omega_0 = 2\pi\,\text{rad/s}$$

$$\xi_0 = 1 \qquad \omega_0 = 2\pi$$

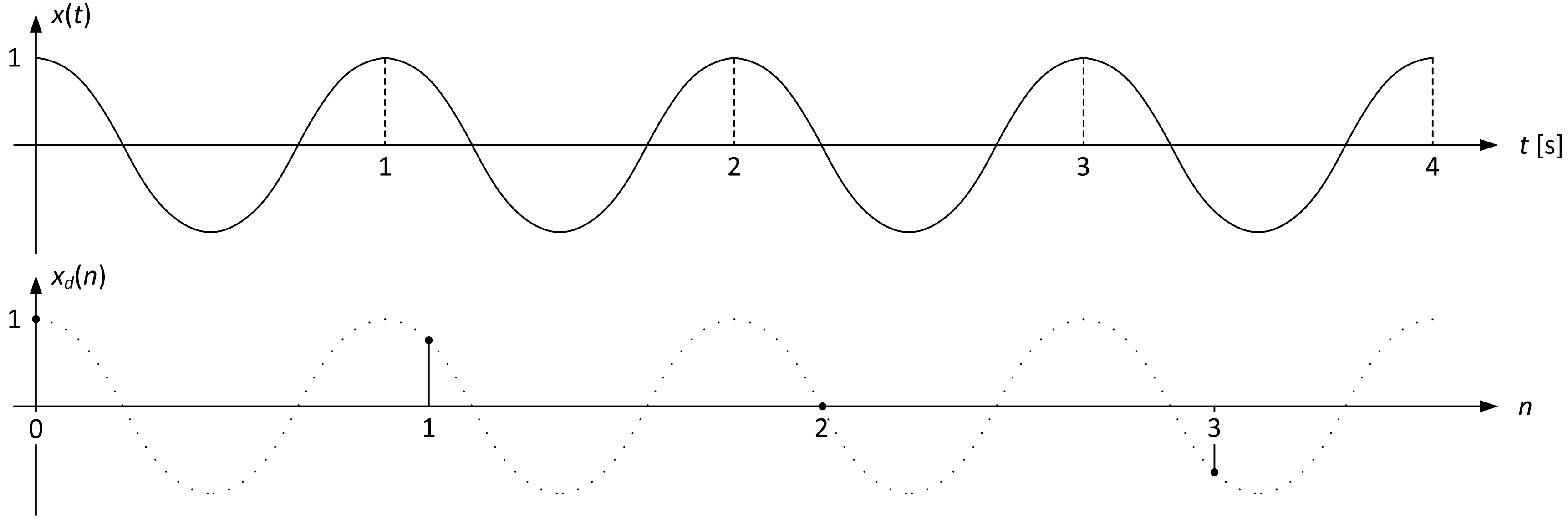


# Examples of sampling

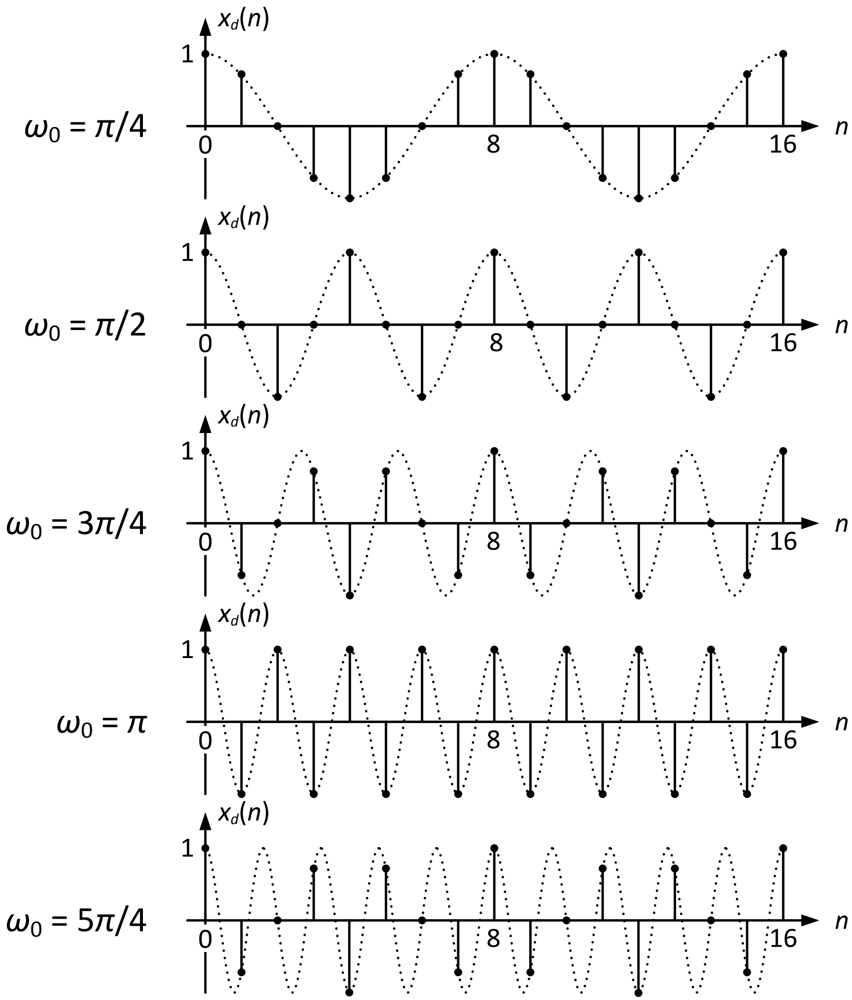


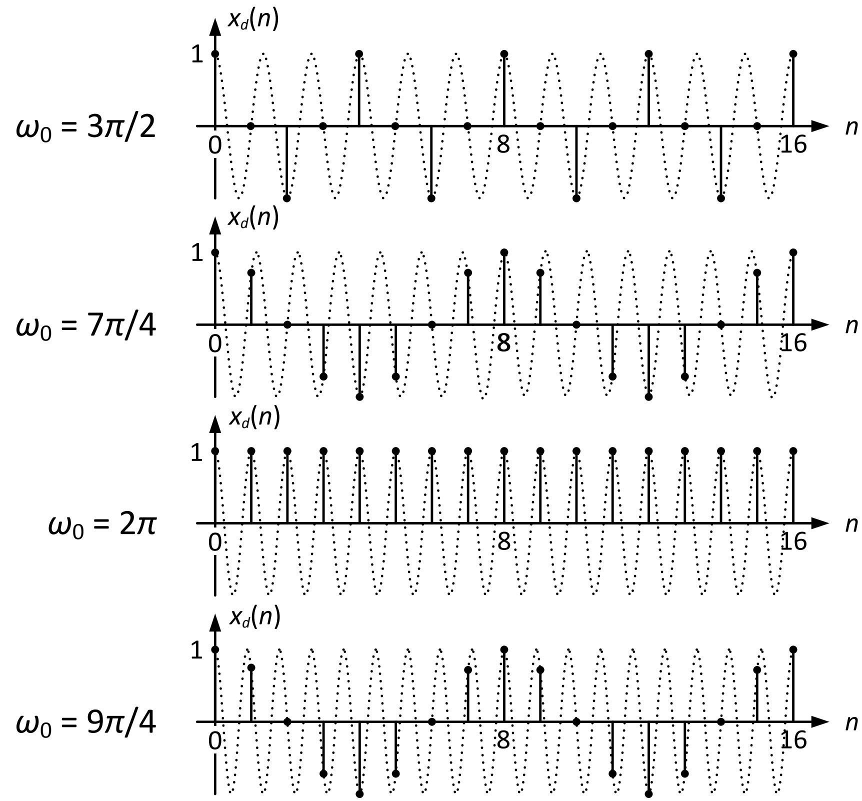

$$f_0 = 1\,\text{Hz} \quad \Omega_0 = 2\pi\,\text{rad/s}$$

$$\xi_0 = \frac{9}{8} \quad \omega_0 = \frac{9\pi}{4}$$



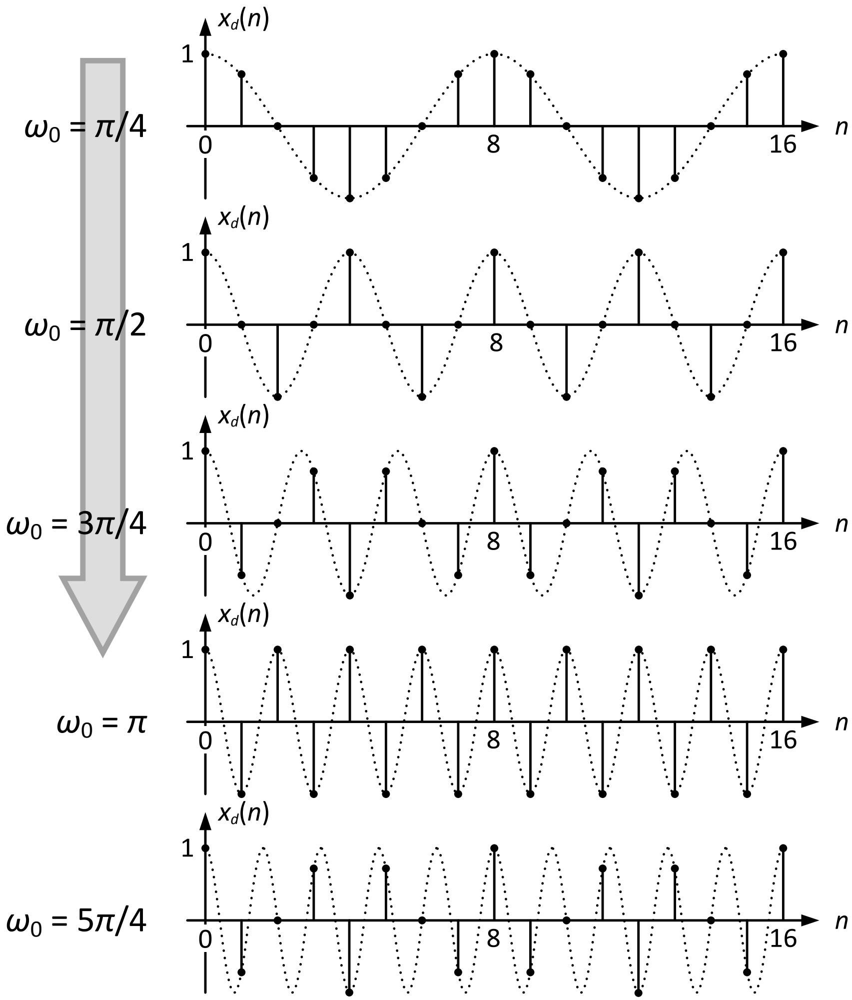
# Frequency of discrete signals

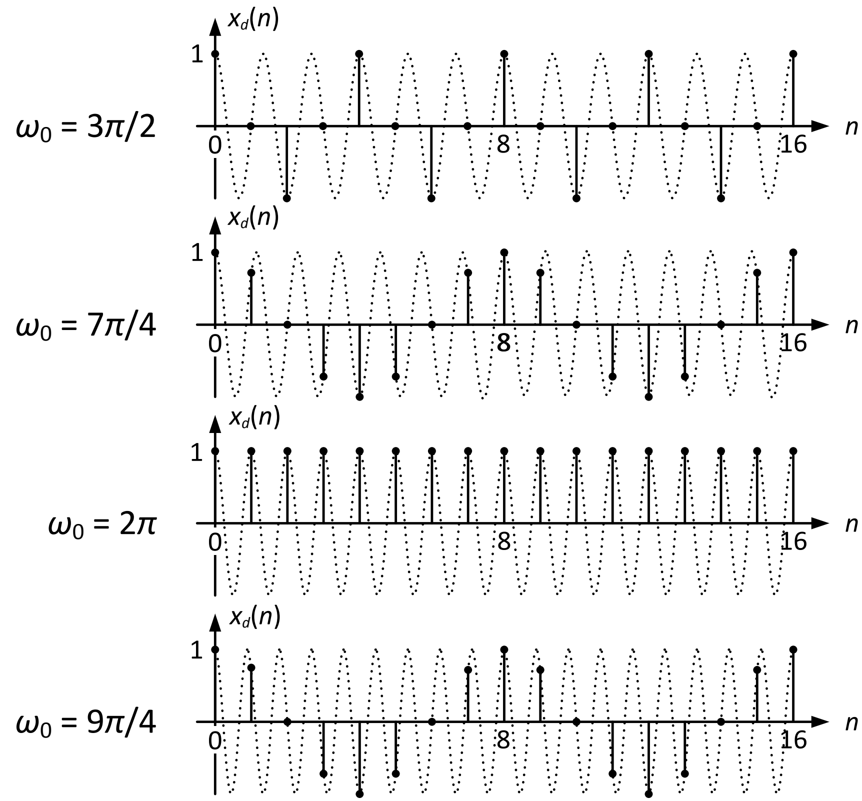



Greater values of $\omega_0$ do not necessarily imply that the discrete-time signal will change more quickly!

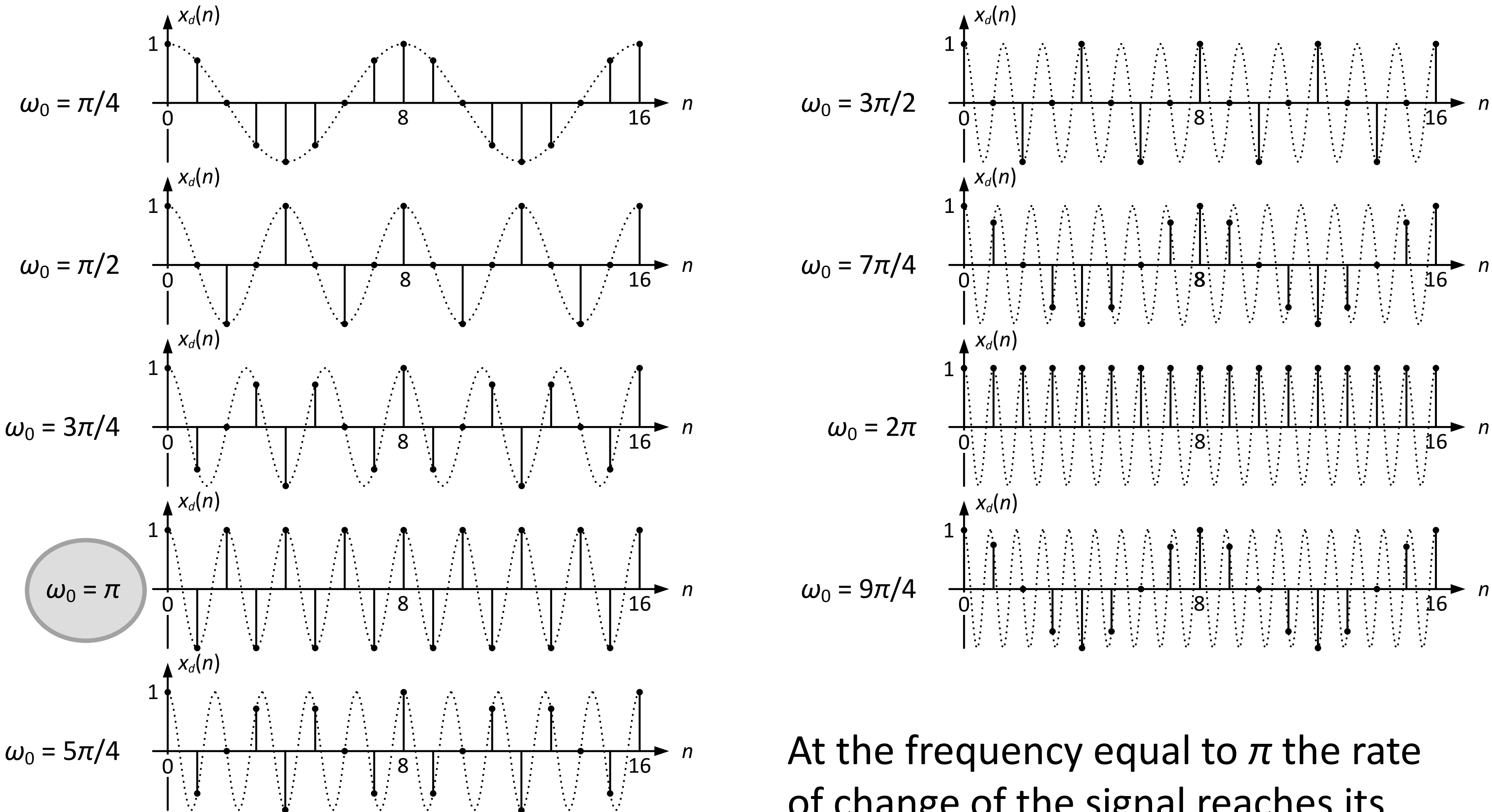
# Frequency of discrete signals



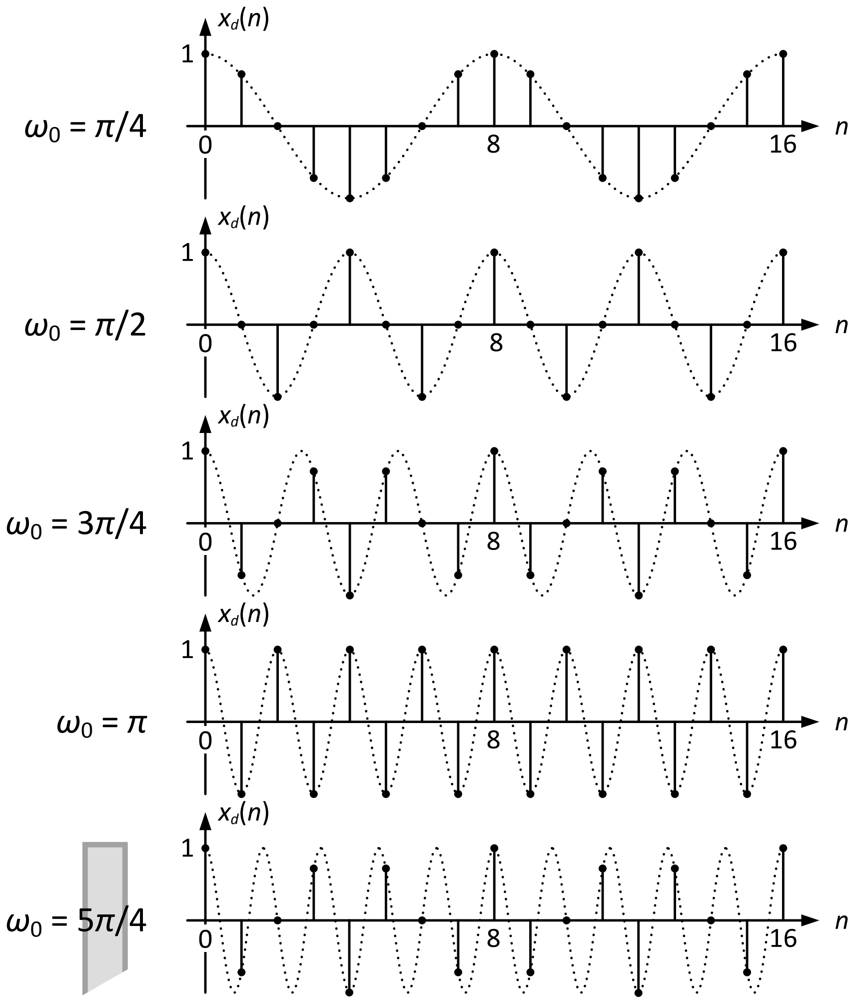

As $\omega_0$ increases from 0 to $\pi$, so does the rate of change of the discrete-time signal

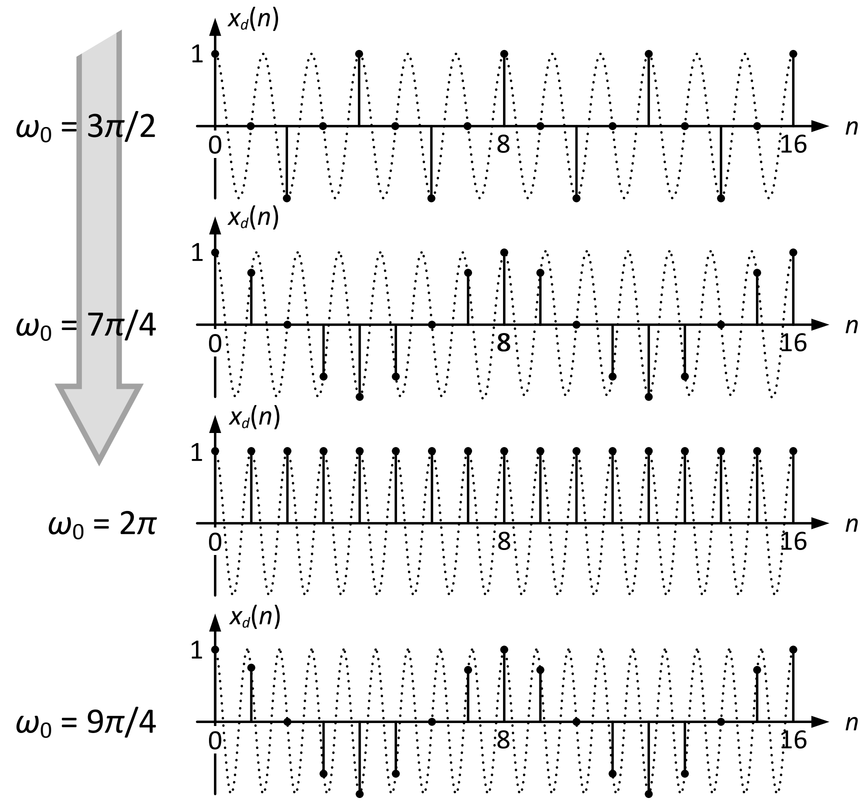
# Frequency of discrete signals




At the frequency equal to $\pi$ the rate of change of the signal reaches its maximum

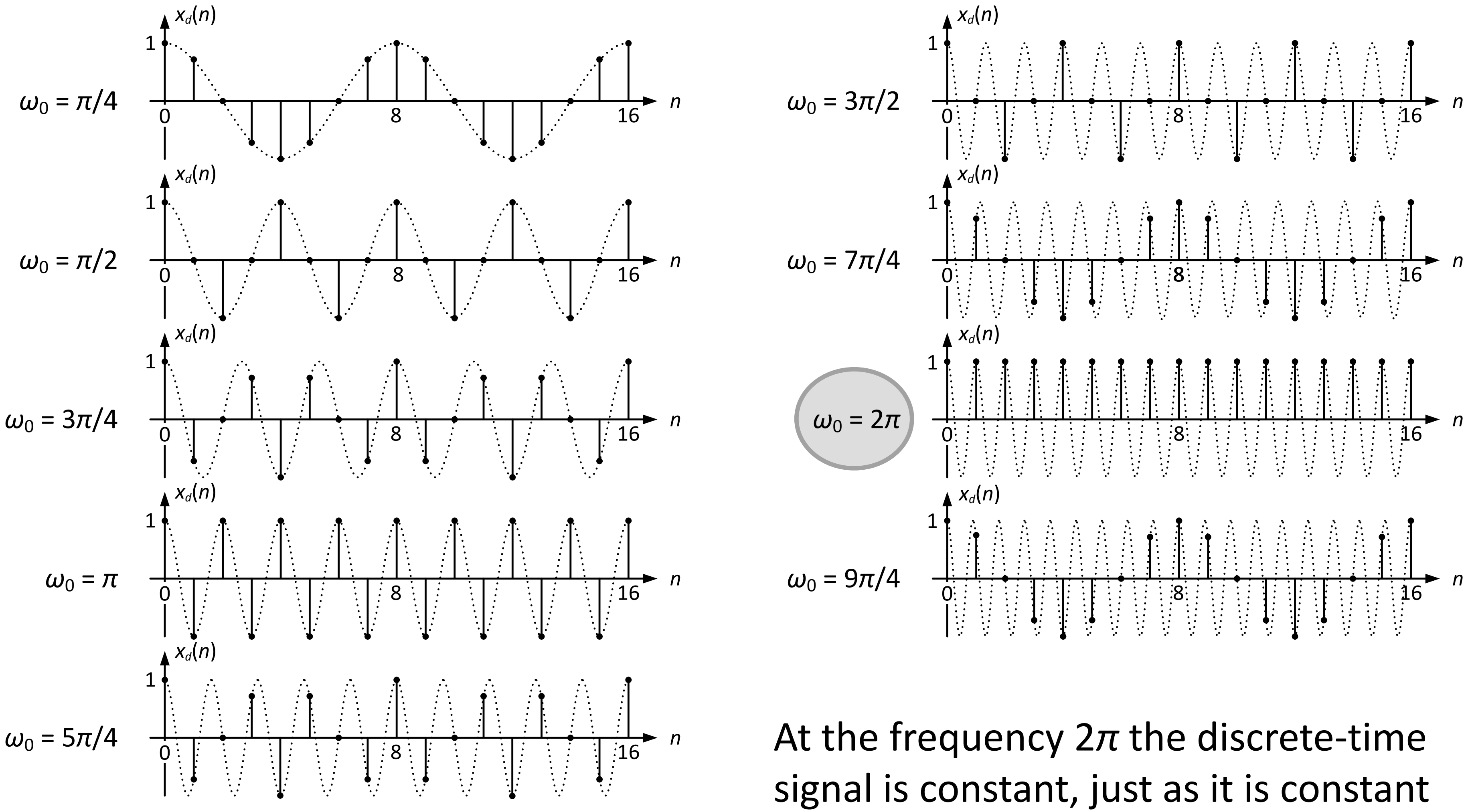
# Frequency of discrete signals



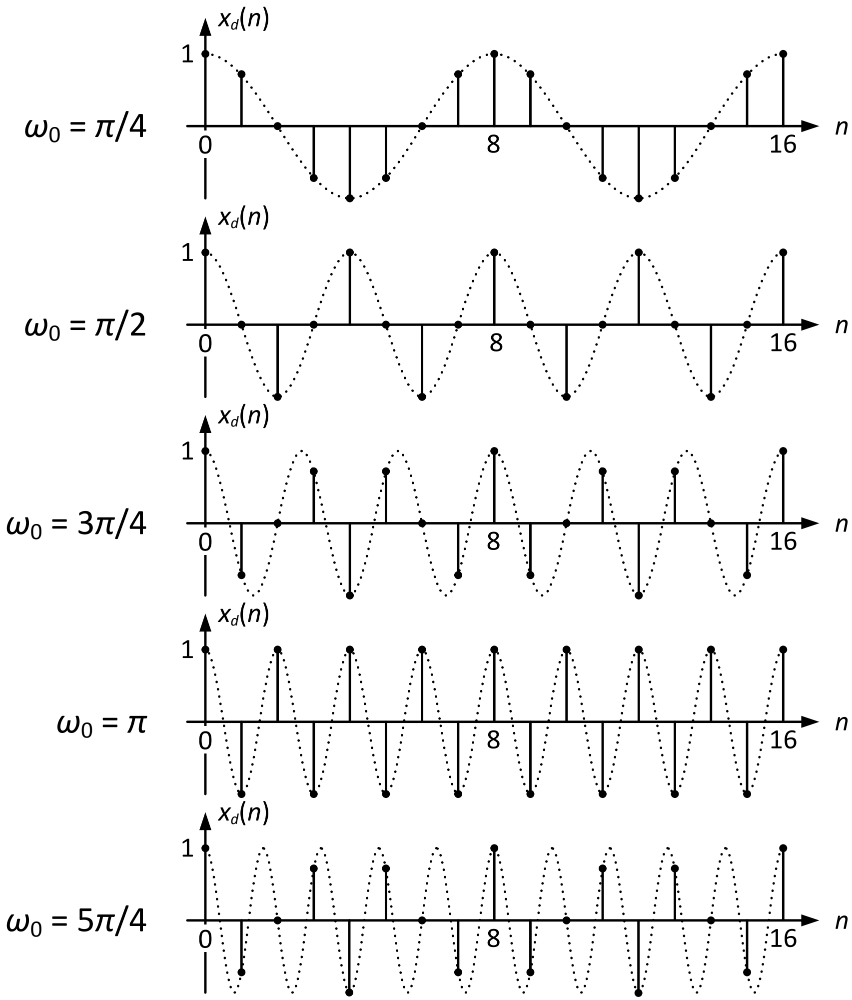

With $\omega_0$ increasing further, the rate of change of the discrete-time signal *decreases*

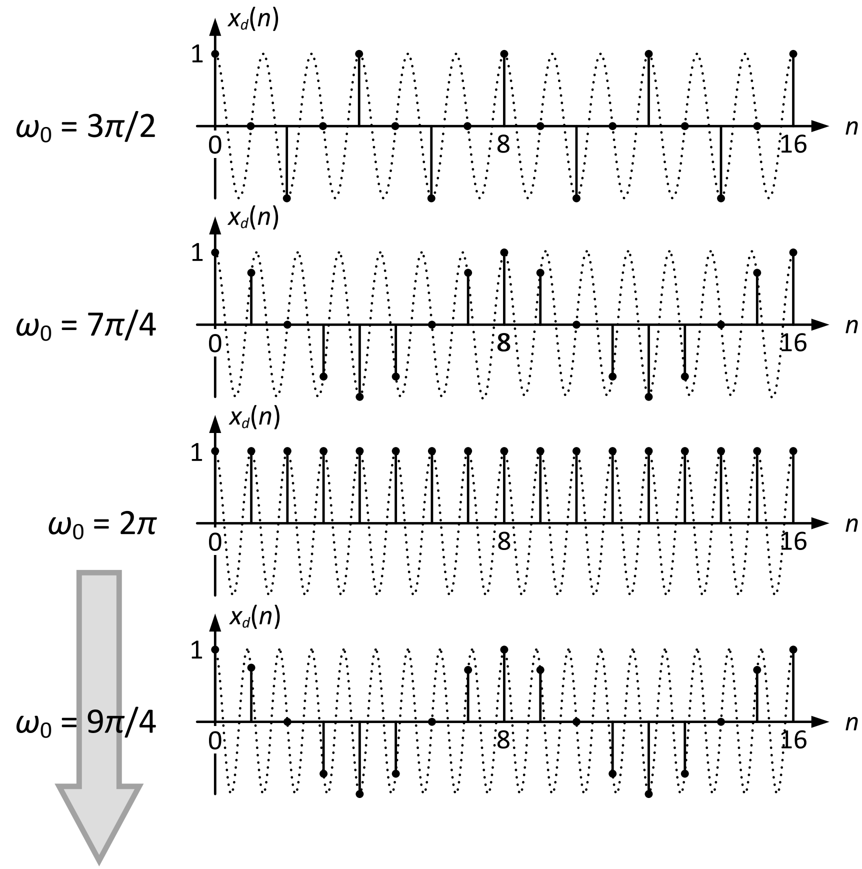
# Frequency of discrete signals




At the frequency $2\pi$ the discrete-time signal is constant, just as it is constant at the frequency 0

# Frequency of discrete signals

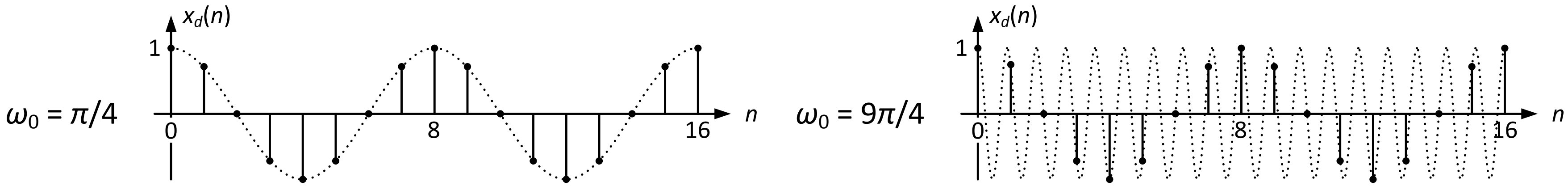



With $\omega_0$ increasing further, the cycle repeats itself with the period equal to $2\pi$

# Frequency of discrete signals

- Signals $\cos(n\pi/4)$ and $\cos(9n\pi/4)$ are identical:




- The same goes for any two sinusoidal discrete-time signals whose frequencies differ by an integer multiple of $2\pi$

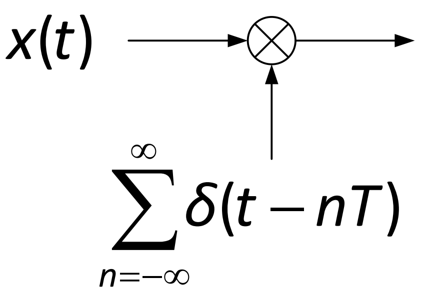
$$e^{j(\omega_0+2k\pi)n} = e^{j\omega_0 n}e^{j2k\pi n} = e^{j\omega_0 n}, k \in \mathbf{Z}$$

1

- In the continuous-time case, two complex sinusoids of different frequencies are always different themselves
  - This is because in the continuous-time case the fundamental period could be just any non-zero *real* number

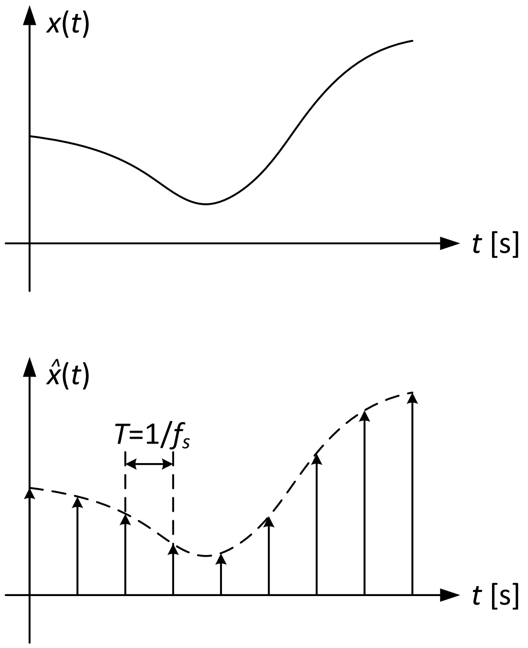
# Sampling

- Process of conversion of a continuous-time signal into a discrete-time signal
- Full digitalization of a signal also requires *quantization*, whereby the *values* of samples also become discrete




$$\hat{x}(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



# Sampling

- Sampling theorem

$$f_s > 2f_{\max} \Leftrightarrow T_s < \frac{T_{\min}}{2}$$

- Hardware limitation

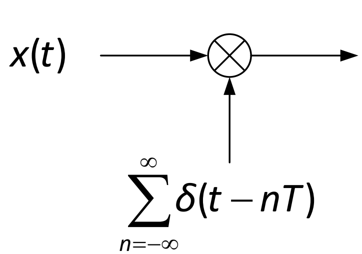
$$f_s \leq f_{proc} \Leftrightarrow T_s \geq T_{proc}$$

| APPLICATION | $f_{\max}$ | $f_s$ |
|---|---|---|
| geophysics | 500 Hz | 1 kHz |
| biomedicine | 1 kHz | 2 kHz |
| mechanics | 2 kHz | 4 kHz |
| speech (telephony) | 4 kHz | 8 kHz |
| audio | 20 kHz | 40 kHz |
| video | 4 MHz | 8 MHz |

- In order for digital signal processing with ideal reconstruction of the original continuous-time signal to be possible, the following must hold:

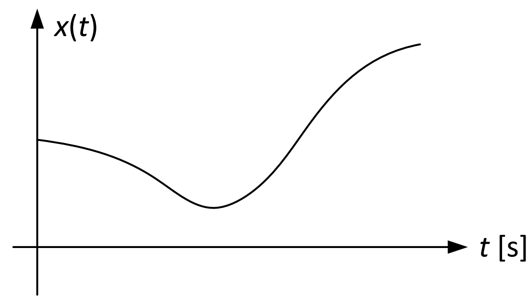
$$2f_{\max} < f_{proc} \Leftrightarrow \frac{T_{\min}}{2} > T_{proc}$$

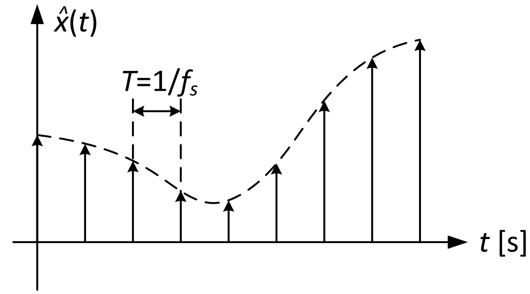
# Spectrum of a sampled signal




$$\hat{x}(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$= \sum_{n=-\infty}^{\infty}x(nT)\delta(t-nT)$$




$$\hat{X}(f) = \mathrm{F}\{\hat{x}(t)\} = \int_{-\infty}^{\infty}\hat{x}(t)e^{-j2\pi ft}dt$$

$$\hat{X}(f) = \int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty}x(nT)\delta(t-nT)e^{-j2\pi ft}dt$$

$$\hat{X}(f) = \sum_{n=-\infty}^{\infty}x(nT)\int_{-\infty}^{\infty}\delta(t-nT)e^{-j2\pi ft}dt$$

$$\hat{X}(f) = \sum_{n=-\infty}^{\infty}x(nT)e^{-j2\pi fnT}\int_{-\infty}^{\infty}\delta(t-nT)dt$$

1

$$\hat{X}(f) = \sum_{n=-\infty}^{\infty}x(nT)e^{-j2\pi fnT}$$

FOURIER TRANSFORM OF THE DISCRETE-TIME SIGNAL $x_d(n) = x(nT)$

# Spectrum of a sampled signal

- The condition for existence is $\left|\hat{X}(f)\right| < \infty$ for each $f$

  Since the following holds:

  $$\left|\hat{X}(f)\right| = \left|\sum_{n=-\infty}^{\infty} x(nT)e^{-j2\pi fnT}\right| \le \sum_{n=-\infty}^{\infty} \left|x(nT)\right|\underbrace{\left|e^{-j2\pi fnT}\right|}_{1} = \sum_{n=-\infty}^{\infty} \left|x(nT)\right|,$$

  a sufficient condition for convergence is $\sum_{n=-\infty}^{\infty} \left|x(nT)\right| < \infty$

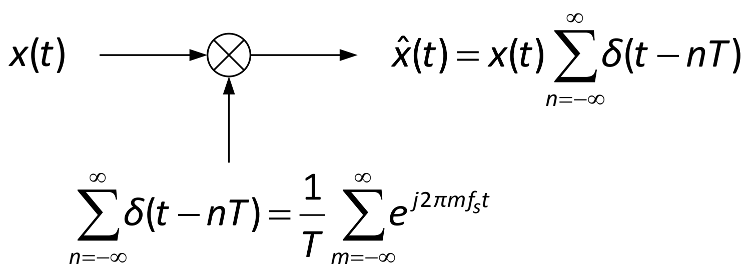
- Spectrum of a sampled signal is *periodic*, with period $f_s$

$$\hat{X}(f + kf_s) = \sum_{n=-\infty}^{\infty} x(nT)e^{-j2\pi(f+kf_s)Tn}$$

$$= \sum_{n=-\infty}^{\infty} x(nT)e^{-j2\pi fTn}\underbrace{e^{-j2\pi kf_sTn}}_{1} = \hat{X}(f)$$

# Spectrum of a sampled signal

- Spectrum of the sampled signal is related to the spectrum of the original continuous-time signal $X(f)$




$$\hat{X}(f) = \int_{-\infty}^{\infty} \hat{x}(t) e^{-j2\pi f t}\, dt$$

$$\hat{X}(f) = \int_{-\infty}^{\infty} x(t) \sum_{n=-\infty}^{\infty} \delta(t-nT) e^{-j2\pi f t}\, dt$$

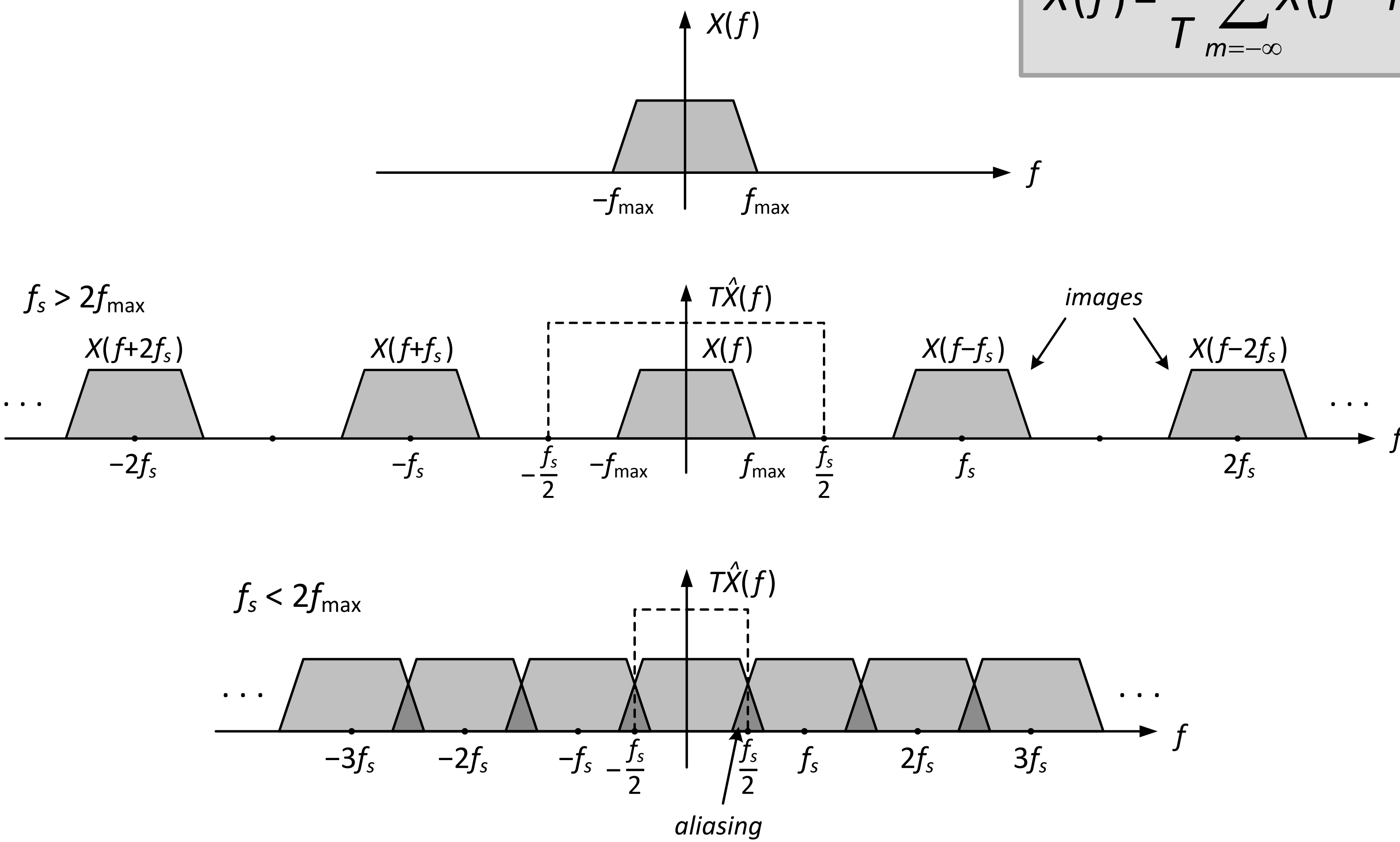
$$\hat{X}(f) = \frac{1}{T}\sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} x(t) e^{j2\pi m f_s t} e^{-j2\pi f t}\, dt$$

$$\hat{X}(f) = \frac{1}{T}\sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} x(t) e^{-j2\pi (f - m f_s) t}\, dt$$

$$\boxed{\hat{X}(f) = \frac{1}{T}\sum_{m=-\infty}^{\infty} X(f - m f_s)}$$

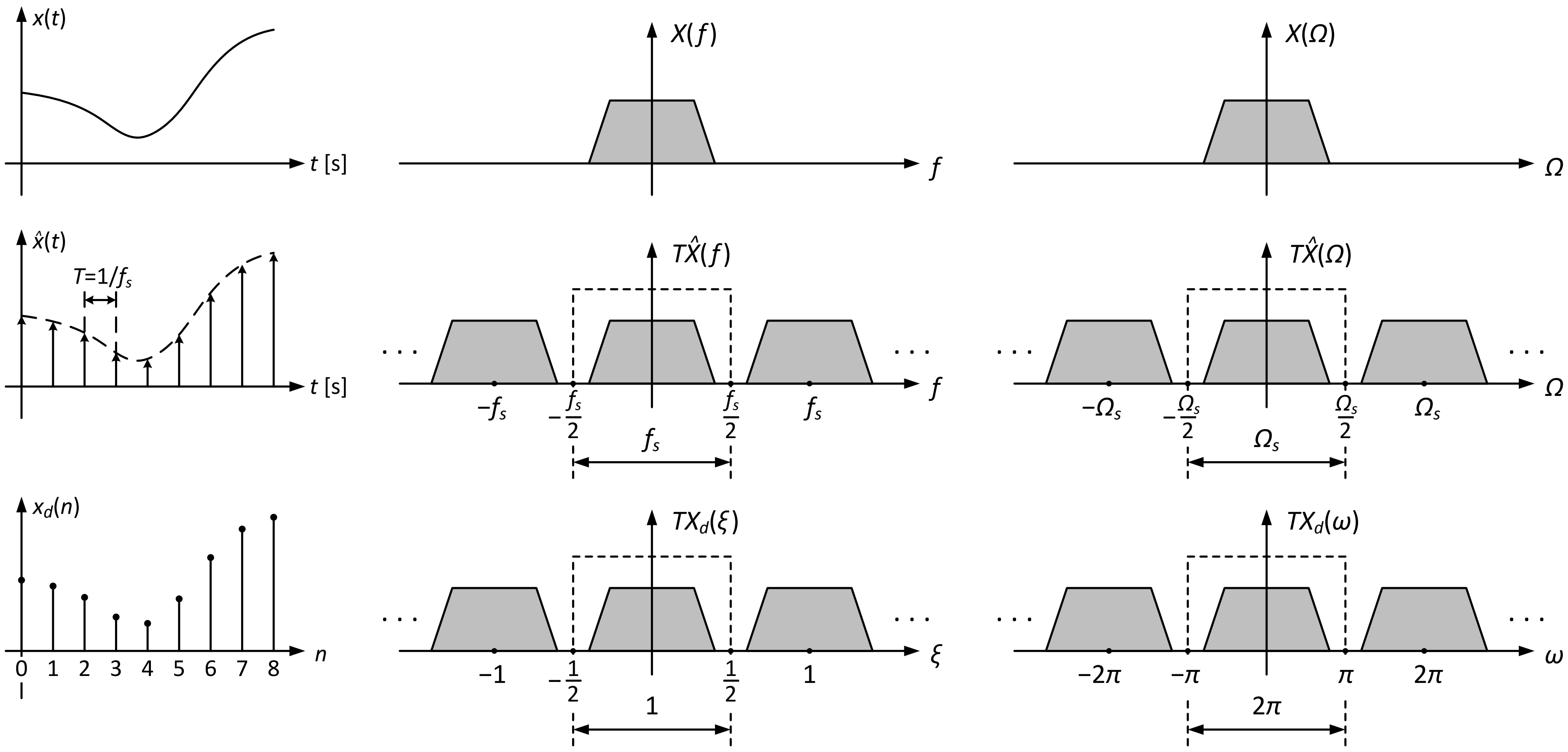
# Spectrum of a sampled signal

$$\hat{X}(f) = \frac{1}{T}\sum_{m=-\infty}^{\infty} X(f - mf_s)$$



# Nyquist interval

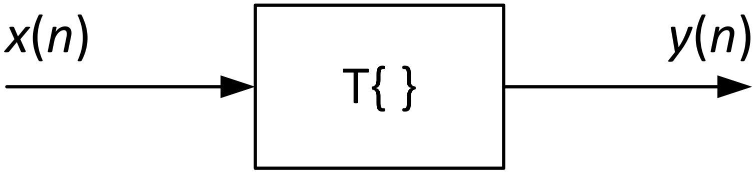
- The fundamental period of the spectrum of a discrete-time signal is called the *Nyquist interval*



# Discrete-time system

- Transform T{} which maps the input signal $x(n)$ (*excitation*) into an output signal $y(n)$ (*response*):

$$T\{x(n)\} = y(n)$$




- From a mathematical standpoint, a discrete-time system is a mapping from the set of discrete-time signals $D_{\mathbf{R}}$ into itself, defined by the operator T{}
  - The general relationship between $x(n)$ and $y(n)$ is called the *input-output relationship*

# Properties of discrete-time systems

**Additivity**

$$\forall x_1(n), x_2(n) \in \mathrm{D}_{\mathbf{R}}$$

$$\mathrm{T}\{x_1(n) + x_2(n)\} = \mathrm{T}\{x_1(n)\} + \mathrm{T}\{x_2(n)\}$$

**Homogeneity**

$$\forall x_1(n) \in \mathrm{D}_{\mathbf{R}},\ \forall a \in \mathbf{R},$$

$$\mathrm{T}\{a x_1(n)\} = a\mathrm{T}\{x_1(n)\}$$

# Properties of discrete-time systems

## Linearity

$$\forall x_1(n), x_2(n) \in \mathrm{D}_{\mathbf{R}},\ \forall a, b \in \mathbf{R},$$

$$\mathrm{T}\{a x_1(n) + b x_2(n)\} = a\mathrm{T}\{x_1(n)\} + b\mathrm{T}\{x_2(n)\}$$

- A system is linear if and only if it is both additive and homogeneous

## Time invariance

$$\forall x(n) \in \mathrm{D}_{\mathbf{R}}, \forall k \in \mathbf{Z},$$

$$\mathrm{T}\{x(n)\} = y(n) \Rightarrow \mathrm{T}\{x(n-k)\} = y(n-k)$$

# Properties of discrete-time systems

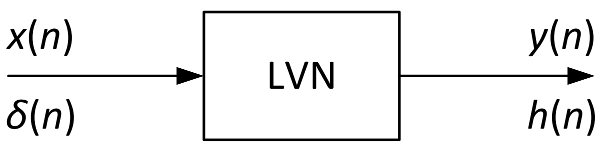
**Causality**

$$\forall x_1(n), x_2(n) \in \mathrm{D}_{\mathbf{R}},\ \forall n_0 \in \mathbf{Z},$$
$$x_1(n) = x_2(n), n \le n_0 \Rightarrow\ y_1(n) = y_2(n), n \le n_0$$

- The system is causal if its response at no time instant *n* depends on the values of excitation in any future time instant (*n*+1, *n*+2,...)
- All discrete-time systems which perform real-time signal processing have to fulfil this condition

# Linear time-invariant systems

- LTI systems have particularly interesting properties




$$y(n) = T\{x(n)\}$$

$$= T\left\{\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right\}$$

ADDITIVITY

$$= \sum_{k=-\infty}^{\infty} T\{x(k)\delta(n-k)\}$$

HOMOGENEITY

$$= \sum_{k=-\infty}^{\infty} x(k)T\{\delta(n-k)\}$$

TIME INVARIANCE

$$= \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$y(n) = x(n) * h(n)$$

# Impulse response

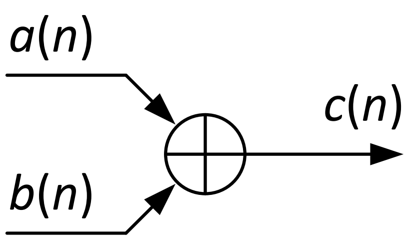
- The response to the $\delta$-impulse, which also uniquely identifies an LTI system
- Properties of the impulse response are related to system properties
  - The impulse response is causal if and only if the system is causal
- LTI systems whose input-output relationship is a linear difference equation with constant coefficients are of particular interest in practice:

$$\sum_{i=0}^{N} a_i y(n-i) = \sum_{i=0}^{M} b_i x(n-i) \qquad a_N \neq 0, a_0 = 1$$

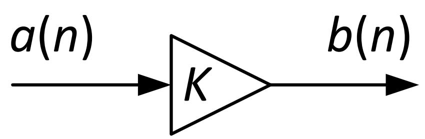
- Input-output relationship in this form also allows us to represent an LTI system graphically

# Graphical representation of LTI systems

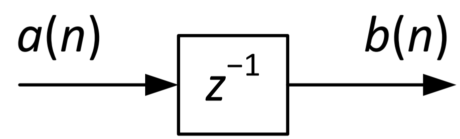
- An LTI system whose IOR is a linear difference equation with constant coefficients can be graphically represented using *adders*, *multipliers* and *time delay units*




$c(n)=a(n)+b(n)$



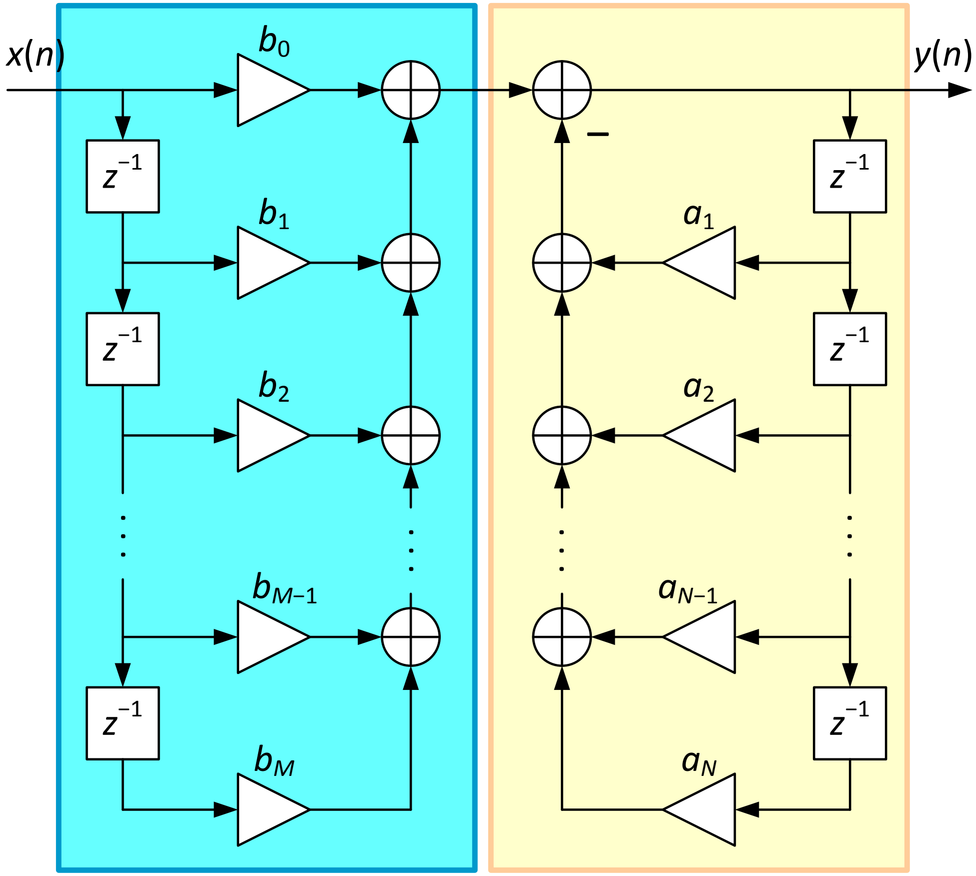

$b(n)=Ka(n)$




$b(n)=a(n-1)$

# Structures for realization of LTI systems

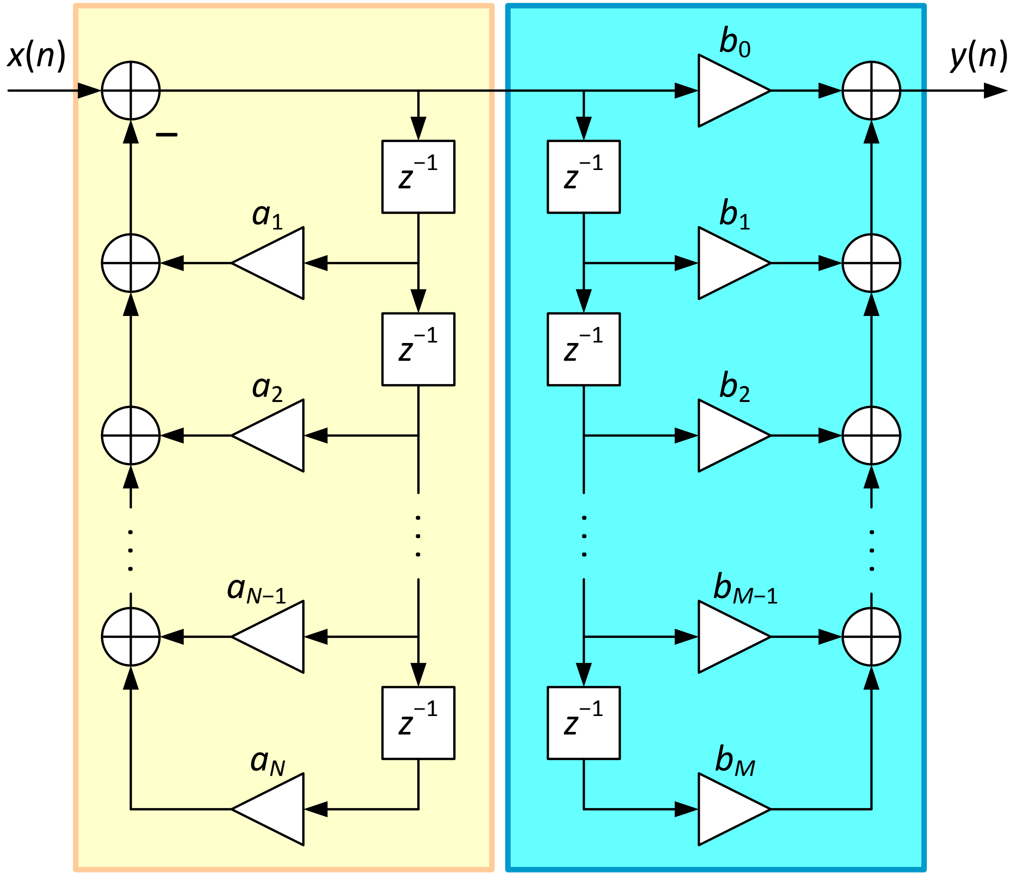
$$y(n) = -\sum_{i=1}^{N} a_i y(n-i) + \sum_{i=0}^{M} b_i x(n-i)$$




DIRECT FORM STRUCTURE OF AN LTI SYSTEM (DIRECT FORM I)

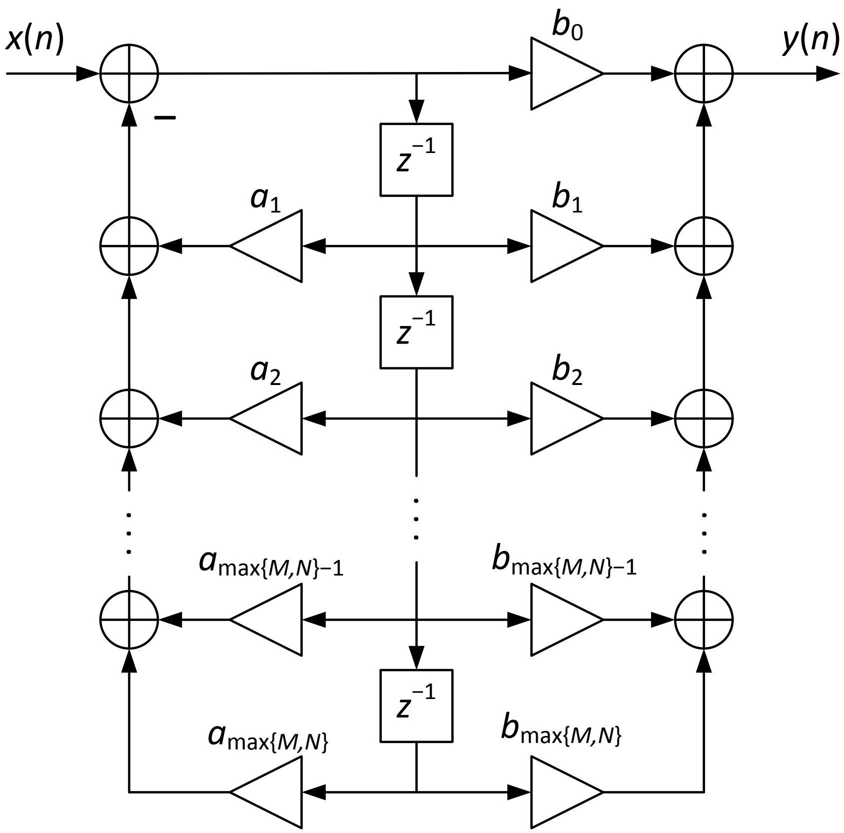
# Structures for realization of LTI systems

$$y(n) = -\sum_{i=1}^{N} a_i y(n-i) + \sum_{i=0}^{M} b_i x(n-i)$$



# Structures for realization of LTI systems

$$y(n) = -\sum_{i=1}^{N} a_i y(n-i) + \sum_{i=0}^{M} b_i x(n-i)$$




DIRECT FORM STRUCTURE OF AN LTI SYSTEM (DIRECT FORM II, CANONICAL)

# FIR systems (Finite Impulse Response)

- IOR has all coefficients $a_i$ equal to 0 (except $a_0 = 1$)

$$y(n) = -\cancelto{0}{\sum_{i=1}^{N} a_i y(n-i)} + \sum_{i=0}^{M} b_i x(n-i)$$

$$y(n) = \sum_{i=0}^{M} b_i x(n-i) \Rightarrow h(n) = \sum_{i=0}^{M} b_i \delta(n-i)$$

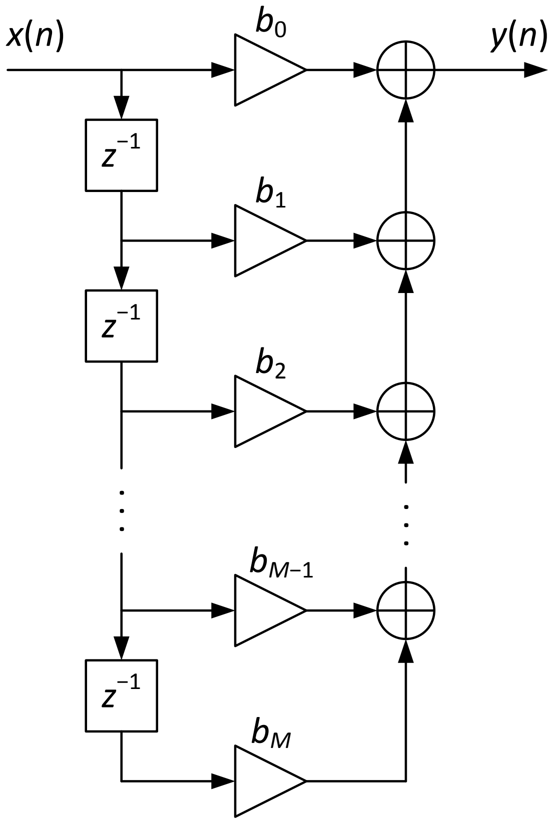
- Coefficients $b_i$ are identical to the values of the samples of the FIR system's impulse response:

$$y(n) = h(0)x(n) + h(1)x(n-1) + \ldots + h(M)x(n-M)$$

- Generally, FIR systems are all LTI systems with impulse response of finite duration, it does not have to start exactly at $n$=0

# Direct form structure of a FIR system

$$y(n) = \sum_{i=0}^{M} b_i x(n-i)$$



# FIR systems (Finite Impulse Response)

Find the IOR of the system with the impulse response:

$$h(n) = 2\delta(n) + 3\delta(n-1) + 3\delta(n-2) + 2\delta(n-3)$$

Find the impulse responses of the systems with the following IORs:

$$y(n) = x(n) + 2x(n-1) + 3x(n-2)$$

$$y(n) = x(n) - x(n-4)$$

# IIR systems (Infinite Impulse Response)

- IOR has at least one non-zero coefficient $a_i$ (except $a_0$)

$$y(n) = -\sum_{i=1}^{N} a_i y(n-i) + \sum_{i=0}^{M} b_i x(n-i)$$

$$h(n) = -\sum_{i=1}^{N} a_i h(n-i) + \sum_{i=0}^{M} b_i \delta(n-i)$$

- Impulse response cannot be directly obtained from IOR coefficients

1) $y(n) = y(n-1) + x(n)$

$h(n) = h(n-1) + \delta(n)$

$(h(n) = u(n))$

2) $y(n) = \alpha y(n-1) + x(n)$

$h(n) = \alpha h(n-1) + \delta(n)$

$(h(n) = \alpha^n u(n))$

- Generally, IIR systems are all LTI systems with impulse response of infinite duration, it does not have to start exactly at $n$=0

# Examples of discrete systems

Examine the following systems with respect to their linearity, time invariance and causality:

1) $y(n) = 4x(n)$
2) $y(n) = x(n) + 3x(n-1)$
3) $y(n) = x(n) + 1$
4) $y(n) = x(n^2)$
5) $y(n) = x^2(n)$
6) $y(n) = x(2n)$
7) $y(n) = x(n-1)x(n+1)$
8) $y(n) = |x(n)|$
9) $y(n) = x(n)u(n)$
10) $y(n) = \max\{x(n+1), x(n), x(n-1)\}$
11) $y(n) = nx(n)$