

## Domaći iz integrala

**Primer 1.**

$$\text{a) } \int (2x^4 - x^3 - 2) dx; \quad \text{b) } \int (\sin x + e^x) dx; \quad \text{c) } \int (\sqrt{x} + 1)(x - \sqrt{x} + 1) dx; \quad \text{d) } \int \frac{\sqrt{x} - x^3 e^x + x^2}{x^3} dx.$$

**Primer 2.**

$$\text{a) } \int \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x} dx; \quad \text{b) } \int \frac{1 + \cos^2 x}{1 + \cos 2x} dx; \quad \text{c) } \int \operatorname{tg}^2 x; \quad \text{d) } \int \frac{(1 + x^2) dx}{x(1 + x^2)};$$

$$\text{e) } \int \frac{dx}{\cos 2x + \sin^2 x}; \quad \text{f) } \int (\arcsin x + \arccos x) dx; \quad \text{g) } \int \frac{\sqrt{\ln x}}{x}; \quad \text{i) } \int \frac{(\operatorname{arctg} x)^2 dx}{1 + x^2};$$

$$\text{i) } \int \frac{(\operatorname{arctg} x)^2 dx}{1 + x^2}; \quad \text{j) } \int \frac{1}{(\arcsin x)^2 \sqrt{1 - x^2}}; \quad \text{k) } \int e^x \sin e^x dx; \quad \text{l) } \int \frac{dx}{\sqrt{1 - 9x^2}};$$

$$\text{m) } \int \frac{dx}{1 + 25x^2}; \quad \text{n) } \int \frac{dx}{9 - 25x^2}; \quad \text{o) } \int \frac{x^2 dx}{x^6 + 4}; \quad \text{p) } \int \frac{e^x}{e^{2x} + 4} dx.$$

**Primer 3.**

$$\text{a) } \int \frac{e^{2x} - 1}{e^x} dx; \quad \text{b) } \int \sqrt{\frac{1-x}{1+x}} dx; \quad \text{c) } \int \frac{2x - \sqrt{\arcsin x}}{\sqrt{1-x^2}} dx; \quad \text{d) } \int \frac{x}{x+4} dx;$$

$$\text{e) } \int \frac{x}{x+4} dx; \quad \text{f) } \int \frac{x}{x+3} dx; \quad \text{g) } \int \frac{(1+x)^2}{x^2+1}; \quad \text{f) } \int \frac{dx}{x^2 - 7x + 1};$$

$$\text{h) } \int \frac{dx}{4x^2 + 4x + 5}; \quad \text{i) } \int \frac{dx}{\sqrt{4x - 3 - x^2}}; \quad \text{j) } \int \frac{dx}{x^2 + x + 1}; \quad \text{k) } \int \frac{dx}{\sqrt{x^2 + x + 1}};$$

**Primer 4.**

$$\text{a) } \int x e^x dx; \quad \text{b) } \int x \sin x dx; \quad \text{c) } \int x^3 \ln x dx; \quad \text{d) } \int \frac{\ln x dx}{x^3}; \quad \text{e) } \int \ln x dx;$$

$$\text{f) } \int \ln(x^2 + 1) dx; \quad \text{g) } \int x^3 e^{2x}; \quad \text{h) } \int e^x \sin x dx; \quad \text{i) } \int e^{\sqrt{x}} dx; \quad \text{j) } \int \frac{x \arcsin x}{\sqrt{(1-x^2)^3}} dx.$$

**Primer 5.**

$$\text{a) } \int \frac{2 dx}{x^2 - 1}; \quad \text{b) } \int \frac{x dx}{(x+1)(2x+1)}; \quad \text{c) } \int \frac{dx}{6x^3 - 7x^2 - 3x}; \quad \text{d) } \int \frac{(x^3 - 2x - 35) dx}{x^2 - 2x - 15};$$

$$\text{e) } \int \frac{x dx}{x^4 - 3x^2 + x + 2}; \quad \text{f) } \int \frac{(x+3) dx}{x^4 - 5x^2 + 4}; \quad \text{g) } \int \frac{(x^2 + 1) dx}{(x-1)^3}; \quad \text{h) } \int \frac{(2x^2 - 4x + 3) dx}{x^4 - 6x^3 + 13x^2 - 12x + 4};$$

$$\text{i) } \int \frac{x^3 - 6x^2 + 9x + 7}{(x-2)^2(x-5)} dx; \quad \text{j) } \int \frac{x^5 dx}{(x-1)^2(x^2-1)}; \quad \text{k) } \int \frac{(7x^3 - 9) dx}{x^4 - 5x^3 + 6x^2}; \quad \text{l) } \int \frac{x^4 + 1}{x^3 - x^2 + x - 1} dx;$$

$$\text{m) } \int \frac{x^2 - x - 21}{2x^3 - x^2 + 8x - 4} dx; \quad \text{n) } \int \frac{5x^3 - 3x^2 + 7x - 3}{(x^2 + 1)^2} dx; \quad \text{o)* } \int \frac{dx}{1 + x^4} \quad \text{p) } \int \frac{(x^3 - 6) dx}{x^4 + 6x^2 + 8}.$$

**Primer 6.**

$$\text{a) } \int \sin^2 x dx; \quad \text{b) } \int \sin^4 2x dx; \quad \text{c) } \int \cos^2 x \sin^3 x dx; \quad \text{d) } \int \cos^5 x dx; \quad \text{e) } \int \cos^6 \frac{x}{2} dx; \quad \text{f) } \int \sin^3 x dx;$$

$$\text{g) } \int \frac{dx}{\sin x}; \quad \text{h) } \int \sin 3x \sin 2x dx; \quad \text{i) } \int \sqrt{1 - \cos x} dx; \quad \text{j) } \int \frac{dx}{\sin^3 x}; \quad \text{k) } \int \frac{\cos^3 dx}{\sin^4}; \quad \text{m) } \int \operatorname{tg}^4 x dx.$$

$$\text{n) } \int \frac{dx}{1 + \sin x - \cos x}; \quad \text{o) } \int \frac{dx}{2 + \sin x}; \quad \text{p) } \int \frac{dx}{1 + \sin x + \cos x}; \quad \text{q) } \int \frac{\sin x dx}{4 \sin x + 3 \cos x}.$$

**Primer 7.**

$$\text{a) } \int \frac{2 dx}{\sqrt{x+1}}; \quad \text{b) } \int \frac{dx}{\sqrt{x+1}+1}; \quad \text{c) } \int \frac{x\sqrt{x} dx}{\sqrt{x+x}}; \quad \text{d) } \int \frac{\sqrt[4]{x+1} - 1}{\sqrt[4]{x+1} + 1} dx;$$

$$\text{e) } \int \frac{dx}{\sqrt[3]{2x+3}+1}; \quad \text{f) } \int \frac{\sqrt{x} dx}{\sqrt[3]{x}-1}; \quad \text{g) } \int \frac{\cos(\sqrt{x}) dx}{\sqrt{x}}; \quad \text{h) } \int \sqrt{e^x - 1} dx.$$

**Primer 8.**

$$\text{a) } \int \frac{x^2 + x + 3}{\sqrt{x^2 + 1}} dx; \quad \text{b) } \int \frac{x^2 + x + 2}{\sqrt{x^2 + x + 1}} dx; \quad \text{c) } \int \sqrt{x^2 + 1} dx; \quad \text{d) } \int \frac{x^3 + 1}{\sqrt{x^2 + x - 1}} dx;$$

$$\text{e) } \int \frac{x^3 + 3x}{\sqrt{5 - x^4 - 2x^2}} dx; \quad \text{f) } \int \frac{e^{3x} + e^{2x} + e^x}{\sqrt{e^{2x} - 1}} dx; \quad \text{g) } \int \frac{\sin^5 x + 2 \sin^3 x - 3 \sin x}{\sqrt{\cos^2 x + \cos x - 1}} dx;$$

**Primer 9.**

$$\text{a) } \int \frac{\sqrt{x} dx}{(1 + \sqrt{x})^2}; \quad \text{b) } \int \frac{x dx}{\sqrt{1 + \sqrt[3]{x}}}; \quad \text{c) } \int \frac{dx}{x^6 \sqrt{x^2 - 1}}; \quad \text{d) } \int \sqrt{16 - x^2} dx;$$

$$\text{e) } \int \frac{dx}{x^2 \sqrt{x^2 + 9}}; \quad \text{f) } \int \frac{\sqrt{x^2 + 1}}{x} dx; \quad \text{g) } \int \frac{\sqrt{x^2 - 4}}{x^2} dx; \quad \text{h) } \int \sqrt[3]{1 + \sqrt[4]{x}} dx;$$

**Primer 10.**

$$\text{a) } \int \frac{dx}{x - \sqrt{x^2 - 1}}; \quad \text{b) } \int \sin \sqrt{x} dx; \quad \text{c) } \int \frac{x^4 dx}{\sqrt{x^2 + 1}}; \quad \text{d) } \int \frac{dx}{1 + \sin^2 x};$$

$$\text{e) } \int \frac{\sqrt{2x + 1}}{x^2} dx; \quad \text{f) } \int \frac{dx}{x^3 \sqrt{(1 + x)^3}}; \quad \text{g) } \int \sqrt{x^2 - 2x - 1}; \quad \text{h) } \int \frac{dx}{1 - x^4};$$

$$\text{i) } \int \sin^n x; \quad \text{j) } \int \cos^n x; \quad \text{k) } \int \frac{dx}{\sin^n x}; \quad \text{l) } \int \frac{dx}{\cos^n x};$$

**Primer 11.**

$$\text{a) } \int_3^6 \sqrt{x - 3} dx; \quad \text{b) } \int_{-1}^1 \frac{x^2 dx}{x - 2}; \quad \text{c) } \int_0^e \frac{\cos(\ln x) dx}{x};$$

$$\text{d) } \int_{\pi/6}^{\pi/2} \cos x \cdot \cot^2 x dx; \quad \text{e) } \int_0^1 \frac{e^x dx}{4 + e^{2x}}; \quad \text{f) } \int_{\sqrt{3}}^{\sqrt{5}} \frac{dx}{x \sqrt{x^4 - 9}}; \quad \text{g) } \int_1^2 x \ln x dx$$

$$\text{h) } \int_2^7 x e^{-5x} dx; \quad \text{i) } \int_0^{\pi/2} x^2 \sin(3x) dx; \quad \text{j) } \int_1^2 x \ln x dx$$

$$I_n = \int_0^{\pi/2} \cos^n x dx; \quad \text{j) } J_n = \int_0^{\pi/2} \sin^n x dx$$

**Primer 12.**

- a) Odrediti površinu ograničenu krivom  $y = \ln x$ ,  $x$ -osom i pravom  $x = e$ .
- b) Odrediti površinu ograničenu krivom  $y = x^2 - 4$  i  $x$ -osom.
- c) Odrediti površinu ograničenu sinusoidom  $y = \sin x$  i intervalom  $[0, 2\pi]$  na  $x$ -osi
- d) Odrediti površinu ograničenu sa krivom  $y = x^3$  i pravama  $y = 6 + x$  i  $2y + x = 0$
- e) Odrediti površinu ograničenu krivim linijama  $y = x^2$  i  $y = \sqrt{x}$
- f) Odrediti površinu ograničenu parabolama  $y = x^2 - 2x$  i  $y = 6x - x^2$
- g) Odrediti zapreminu tela koje nastaje obrtanjem površine ograničene parabolom  $y = x^2 + 2$  i pravom linijom  $y = \frac{x}{2} + 1$  nad  $[0, 1]$  oko  $x$ -ose.
- h) Odrediti zapreminu tela koje nastaje obrtanjem površine ograničene krivom  $y = \frac{1}{8}x^3$  i pravom  $y = 2x$  oko  $y$ -ose.
- i) Odrediti zapreminu torusa koji se dobija obrtanjem kružnice  $x^2 + (y - a)^2 = r^2$ , oko  $x$ -ose, ako je  $0 < r < a$ .
- j) Odrediti dužinu luka krive  $f(x) = 3x^{2/3} - 10$  od tačke  $A(8, 2)$  do tačke  $B(27, 17)$ .
- k) Odrediti dužinu luka krive  $f(x) = \ln x$  od tačke  $A(1, 0)$  do tačke  $B(\sqrt{3}, \frac{\ln 3}{2})$ .
- l) Proveriti da je površina lopte poluprečnika  $a$  jednaka  $M = 4\pi a^2$ .
- m) Odrediti površinu omotača tela koje nastaje rotacijom oko  $y$ -ose krive  $x = y^3$  od tačke  $x = 0$  do tačke  $x = 8$ .