1 (b) $f(x) = \dfrac{4x}{4-x^2}$, $D_f = \mathbb{R} \setminus \{-2, 2\}$

$$f(x) = \frac{4x}{(2-x)(2+x)}$$

Funkcija f nije ni parna, ni neparna, ni periodična.

$f(x) = 0 \iff x = 0$

	-2		0		2	
x	$-$	$-$	0	$+$		$+$
$2-x$	$+$	$+$		$+$	0	$-$
$2+x$	$-$	0 $+$		$+$		$+$
$f(x)$	$+$	$\lightning$ $-$	0	$+$	$\lightning$	$-$

$f(x) > 0$ za $x \in (-\infty, -2) \cup (0, 2)$

$f(x) < 0$ za $x \in (-2, 0) \cup (2, \infty)$

$f(x) = 0$ za $x = 0$

Ekstremne vrijednosti

$$f'(x) = \frac{4 \cdot (4-x^2) - 4x((-2)x)}{(4-x^2)^2} = \frac{16 - 4 \cdot x^2 + 8 \cdot x^2}{(4-x^2)^2}$$

$$f'(x) = \frac{4x^2 + 16}{(4-x^2)^2} = \frac{4(x^2+4)}{(4-x^2)^2}$$

$$f'(x) = \frac{4(x^2+4)}{(4-x^2)^2} > 0 \text{ za sve } x \in D_f$$

Sledi da je f rastuća na svom domenu ($f \uparrow$)

$$f'(x) = \frac{4x^2+16}{(4-x^2)^2}$$

$$f''(x) = \frac{8x\cdot(4-x^2)^2 - (4x^2+16)\cdot 2(4-x^2)\cdot(-2x)}{(4-x^2)^4}$$

$$f''(x) = \frac{8x\cdot(16-8x^2+x^4) + 4x\cdot(4-x^2)(4x^2+16)}{(4-x^2)^4}$$

$$x) = \frac{128\cdot x - 64\cdot x^3 + 8x^5 + 4x\cdot(16\cdot x^2 + 64 - 4x^4 - 16x^2)}{(4-x^2)^4}$$

$$) = \frac{128\cdot x - \cancel{64\cdot x^3} + 8\cdot x^5 + \cancel{64\cdot x^3} + 256\cdot x - 16x^5 - 64x^3}{(4-x^2)^4}$$

$$x) = \frac{-8\cdot x^5 - 64x^3 + 384\cdot x}{(4-x^2)^4}$$

$$x) = \frac{-8\cdot x\cdot(x^4 + 8x^2 - 48)}{(4-x^2)^4}$$

Na znak i nule drugog izraza utiče brojilac.

$$(x^4+8x^2-48)\cdot(8x)(-1)\ \ldots$$

Asimptote $\quad f(x)=\dfrac{4x}{4-x^2}=\dfrac{4x}{(2-x)(2+x)}$

$$\lim_{x\to 2^-}\frac{4x}{(2-x)(2+x)} = \text{"}\frac{8}{0^+\cdot 4}\text{"} = +\infty$$

$\longrightarrow 2$

$$\lim_{x\to 2^+} f(x) = -\infty$$

$$\lim_{x\to -2^-}\frac{4x}{(2-x)(2+x)} = \text{"}\frac{-8}{4\cdot 0^-}\text{"} = \underline{+\infty}$$

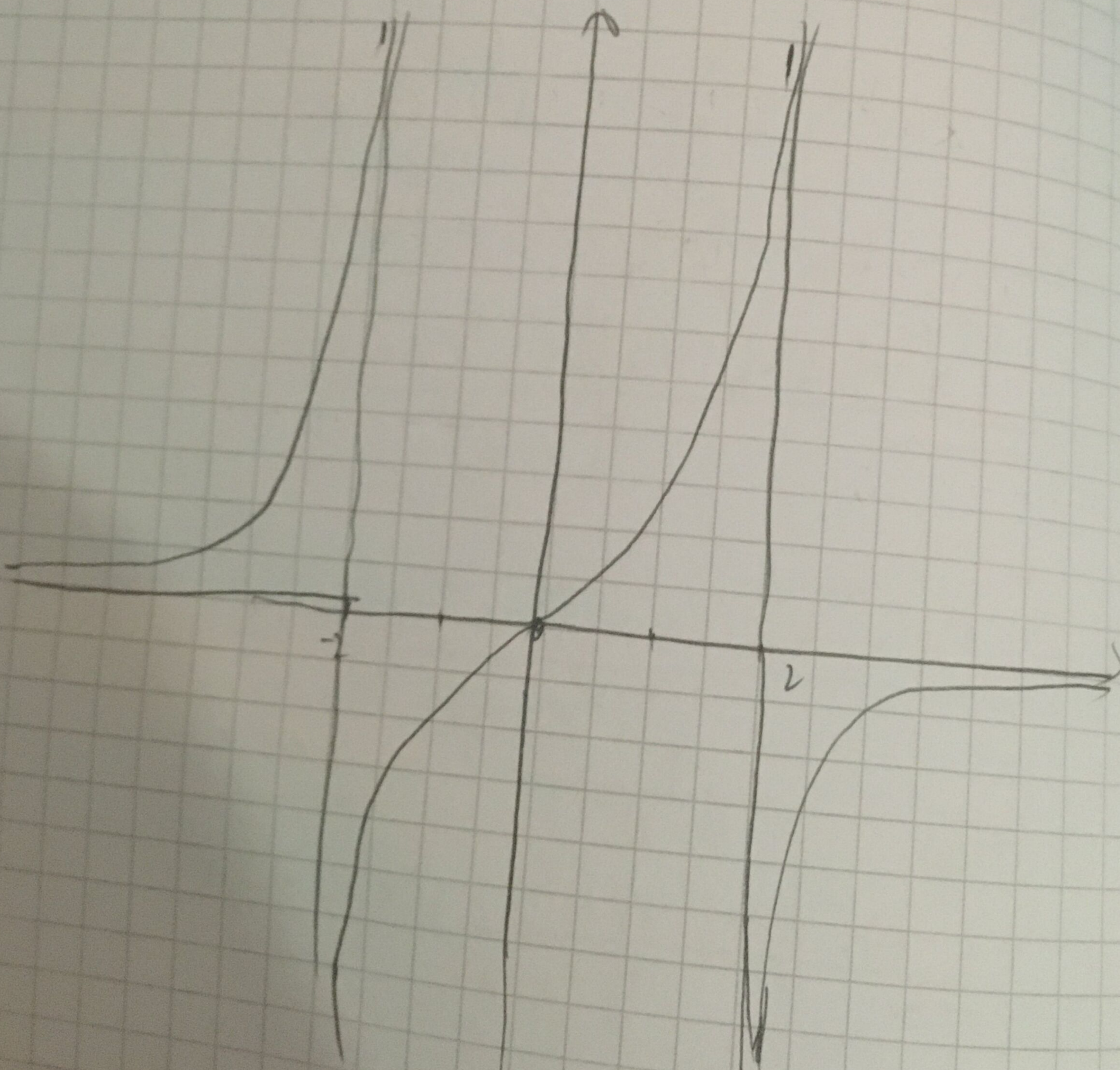
$\longrightarrow -2$

$$\lim_{x\to -2^+} f(x) = -\infty$$

Prave $x=2$ i $x=-2$ su vertikalne asimptote.

$$\lim_{x\to\pm\infty}\frac{4x}{4-x^2}\ ? \qquad \lim_{x\to\pm\infty}\frac{x\cdot 4}{x^{\cancel{2}}\cdot\left(\frac{4}{x^2}-1\right)} = \lim_{x\to\pm\infty}\frac{4}{\left(\frac{4}{x^2}-1\right)x} = 0$$

Prava $y=0$ je horizontalna asimptota



(i) $f(x) = \ln(x^2-1)$

$x^2-1>0$

	-1		1	
$x-1$	$-$		$-$ 0	$+$
$x+1$	$-$ 0	$+$		$+$
x^2-1	$+$		$-$	$+$

za $x \in (-\infty, -1) \cup (1, \infty)$ je $x^2-1>0$, pa je $\ln(x^2-1)$ definisan

$$D_f = (-\infty, -1) \cup (1, \infty)$$

$f(-x) = f(x) \rightarrow$ funkcija je parna
(grafik je simetričan u odnosu na y-osu)

· $f(x)>0$ za $x^2-1>1$

$$x^2-2>0$$

$$(x-\sqrt{2})(x+\sqrt{2})>0$$

	$-\sqrt{2}$		$\sqrt{2}$	
$x-\sqrt{2}$	$-$		$-$ 0	$+$
$x+\sqrt{2}$	$-$ 0	$+$		$+$
$f(x)$	$+$		$-$	$+$

$f(x)>0$ za $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$f(x) < 0$ za $x \in (-\sqrt{2}, \sqrt{2})$

$f(x) = 0$ za $x^2 - 1 = 1$, za $x = -\sqrt{2}$ ili $x = \sqrt{2}$

$$f'(x) = \frac{2x}{x^2-1}$$

$f'(x) = 0$ za $x = 0$, ali $x \notin D_f$!

$f'(x)$ nema nule na D_f

		-1		0		1	
x	$-$		$-$	0	$+$		$+$
$x-1$	$-$		$-$		$-$	0	$+$
$x+1$	$-$	0	$+$		$+$		$+$
$f'(x)$	$-$	$\nexists$	$+$	$\nexists$	$-$	$\nexists$	$+$

$f'(x) < 0$ za $x \in (-\infty, -1)$ $f \downarrow$

$f'(x) > 0$ za $x \in (1, \infty)$ $f \uparrow$

$$f''(x) = \frac{2\cdot(x^2-1) - 2x \cdot 2x}{(x^2-1)^2} = \frac{2x^2 - 2 - 4x^2}{(x^2-1)^2}$$

$$f''(x) = \frac{-2x^2-2}{(x^2-1)^2} = \frac{(-2)(x^2+1)}{(x^2-1)^2} < 0$$

$f''(x) < 0$ $\cap$ (funkcija je konkavna na D_f)

Asimptote $\quad D_f = (-\infty, -1) \cup (1, \infty)$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \ln(x^2-1) = \text{„}\ln 0^+\text{"} = -\infty$$

$$\lim_{x\to -1^-} f(x) = -\infty$$

Prave $x=1$ i $x=-1$ su vertikalne asimptote.

$$\lim_{x\to\pm\infty} \ln(x^2-1) = \pm\infty \rightarrow \text{nema horizontalne asimpt.}$$

Kosa? $\quad y = ax+b$

$$a = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{\ln(x^2-1)}{x} = \text{„}\frac{\infty}{\infty}\text{"}$$

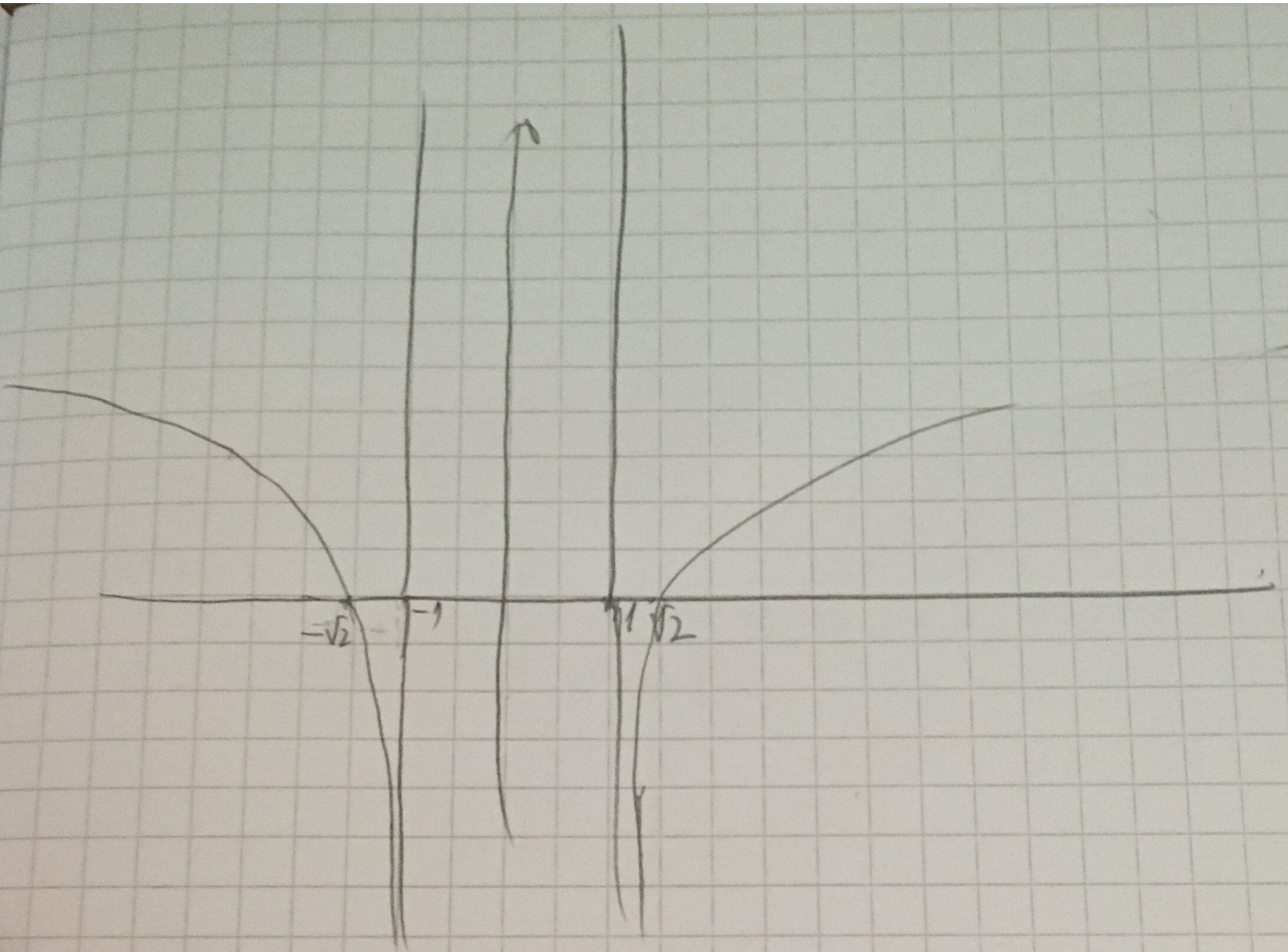
Lopitalovo pravilo:

$$a = \lim_{x\to\pm\infty} \frac{(\ln(x^2-1))'}{x'} = \lim_{x\to\pm\infty} \frac{\frac{2x}{x^2-1}}{1}$$

$$= \lim_{x\to\pm\infty} \frac{2x}{x^2-1} = 0$$

$$b = \lim_{x\to\pm\infty} (f(x) - ax) = \lim_{x\to\pm\infty} f(x) = \pm\infty$$

Nema kose asimptote.



$-\sqrt{2}$
-1
1
$\sqrt{2}$

(e) $y = x^2 e^{-x}$ $D_f = \mathbb{R}$

ni parna ni nep

$f(x)=0 \Leftrightarrow x=0$, $f>0$ svuda

$$f'(x) = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x) = 0$$

$$\Leftrightarrow x=0 \lor x=2$$

$f'(x)=0$ za $x=0$ i $x=2$

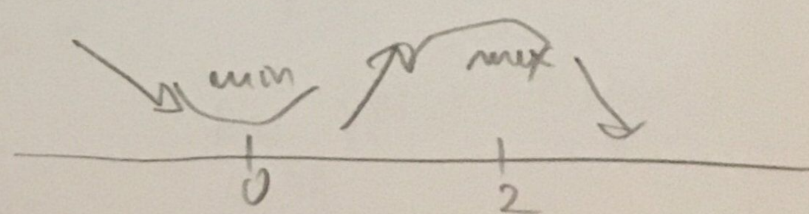
$x(2-x)$?

		0	2	
x	$-$	$+$		$+$
$2-x$	$+$		$+$	$-$
	$-$	$+$		$-$

$f'(x)>0$ za $x \in (0,2)$

$f'(x)<0$ za $x \in (-\infty, 0)$

$\cup\ x \in (2, \infty)$



asimptote:

$\lim_{x\to+\infty} x^2 e^{-x} = \lim_{x\to+\infty} \frac{x^2}{e^x} = \text{"}\frac{+\infty}{+\infty}\text{"} \ ?$

Lop. pravilo $= \lim_{x\to+\infty} \frac{2x}{e^x} = \lim_{x\to+\infty} \frac{2}{e^x} = 0!$

$y=0$ je desna horizontalna asimptota

$\lim_{x\to-\infty} f(x) = +\infty$

$y = ax+b$?

$a = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{x^2 e^{-x}}{x} =$

$\lim_{x\to-\infty} x\cdot e^{-x} = -\infty$

Nema K.A.

$f(2) = 2^2 \cdot e^{-2} = 4e^{-2}$

$f'(x) = x\cdot e^{-x}\cdot(2-x) = (2x-x^2)\cdot e^{-x}$

$f''(x) = (2-2x)\cdot e^{-x} + (2x-x^2)(-1)\, e^{-x}$

$= (2-2x)\, e^{-x} - x(2-x)e^{-x}$

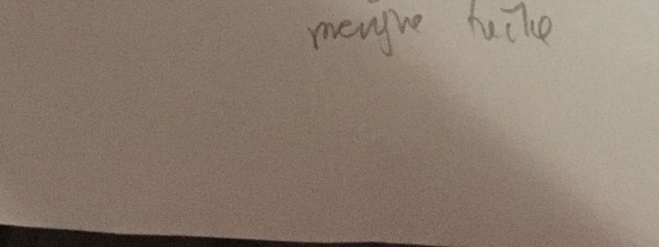
$= e^{-x}(2-2x-2x+x^2)$

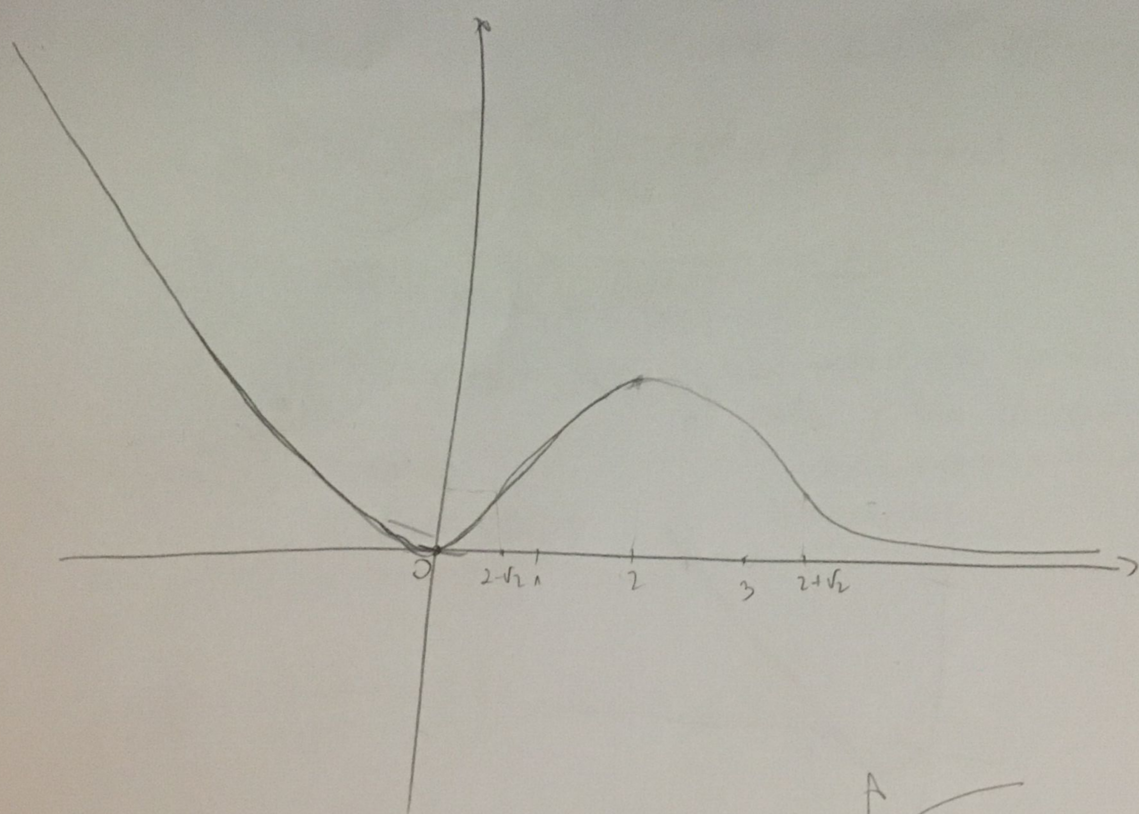
$= e^{-x}(2-4x+x^2)$

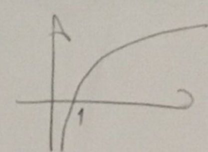
$f''(x)=0 \iff x^2-4x+2=0$

$x_{1,2} = \frac{4\pm\sqrt{16-8}}{2} = \frac{4\pm 2\sqrt{2}}{2}$

$x_{1/2} = 2\pm\sqrt{2}$







(g) $f(x) = x \cdot \ln x$

$\underline{D_f = (0, \infty)}$

$f(x) > 0$?

$f(x) > 0$ za $x \in (1, \infty)$

$f(x) < 0$ za $x \in (0, 1)$, $\underline{\underline{f(1) = 0}}$

$\lim\limits_{x \to 0^+} (x \cdot \ln x) =$ „$0 \cdot -\infty$" ?

$\lim\limits_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim\limits_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{x \to 0^+} (-x) = \underline{0}$ ||

$\boxed{\text{L'Hôpitalovo pravilo}}$

pogoj $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} g(x) = 0$ ali

$c \in \mathbb{R}$
ali
$c = \pm\infty$

$\lim\limits_{x \to c} |f(x)| = \lim\limits_{x \to c} |g(x)| = +\infty$

$\to$ ondu ravnali!

$\lim_{x \to +\infty} x \cdot \ln(x) = +\infty$ nemo h.a.

$\lim_{x \to +\infty} \frac{x \ln x}{x} = +\infty$ nemo k.a.

$f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1 = 0$

$\Leftrightarrow \ln x = -1 \Leftrightarrow x = e^{-1}$

$f'(x) > 0$ se $x > e^{-1}$

$f'(x) < 0$ se $x < e^{-1}$