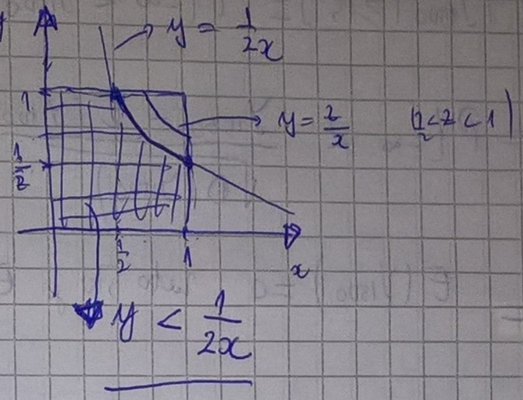
VEROVATNOĆA, 26.9.2017.

(2.) (X,Y), $Z=\max\{XY, \frac{1}{2}\}$



Primetimo da je $P\{Z<\frac{1}{2}\}=0$

$P\{Z=\frac{1}{2}\}=P\{XY<\frac{1}{2}\}$

$y=\frac{1}{2x}$

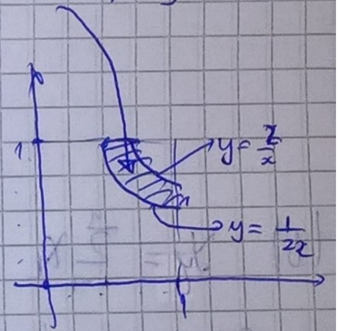
$x=\frac{1}{2}\quad y=1$

- $P\{Z=\frac{1}{2}\}=P\{XY<\frac{1}{2}\}=\int_0^{\frac{1}{2}}\int_0^1 dx\,dy+\int_{\frac{1}{2}}^1\int_0^{\frac{1}{2x}}dx\,dy=\frac{1}{2}+\frac{1}{2}\ln 2$

- Neka je $\frac{1}{2}<z<1$. Tada je

$P\{\frac{1}{2}<Z<z\}=P\{\frac{1}{2}<XY<z\}=P\{\frac{1}{2X}<Y<\frac{z}{X}\}=$

$=\int_{\frac{1}{2}}^{z}\int_{\frac{1}{2x}}^{1}dy\,dx+\int_z^1\int_{\frac{1}{2x}}^{\frac{z}{x}}dy\,dx=z(1-\ln z)-\frac{1}{2}(1+\ln 2)$



- $P\{Z<z\}=1$, za $z>1$.

Sledi da Z nije diskretna slučajna promenljiva.

(5.) X_k – greška zaokruživanja k-tog broja, $k=1,\dots,1500$

X_k su međusobno nezavisne i $X_k: U(-\frac{1}{2},\frac{1}{2})$, $k\in\{1,2,\dots,1500\}$

Neka je $Y_{1500}=\sum_{k=1}^{1500}X_k$ – ukupna greška koja nastaje pri prilikom sabiranja 1500 brojeva

Treba odrediti $P\{|Y_{1500}|\geq 15\}$.

Znamo, ako $X : U(a,b)$, onda $E(X) = \frac{a+b}{2}$, $D(X) = \frac{(b-a)^2}{12}$

$$P\{|Y_{1500}| \geq 15\} = P\{-\infty < Y_{1500} \leq -15\} + P\{15 \leq Y_{1500} < \infty\}$$

$$= P\left\{-\infty < \frac{Y_{1500} - E(Y_{1500})}{\sqrt{D(Y_{1500})}} \leq \frac{-15-E}{\sqrt{D}}\right\} + P\left\{\frac{15-E(Y_{1500})}{\sqrt{D}} \leq \frac{Y_{1500} - E(Y_{1500})}{\sqrt{D(Y_{1500})}} < \infty\right\}$$

$E(Y_{1500}) = 0$ zato što je $E(X_k) = 0$, $k \in \{1, \dots, 1500\}$

$X_k : U\left(-\frac{1}{2}, \frac{1}{2}\right)$

$$D(X_k) = \frac{\left(\frac{1}{2} + \frac{1}{2}\right)^2}{12} = \frac{1}{12}$$

$$\Rightarrow D(Y_{1500}) = \frac{1500}{12}$$

CGT: $$P\{|Y_{1500}| \geq 15\} \approx \Phi\left(\frac{-15}{\sqrt{\frac{1500}{12}}}\right) + \frac{1}{2} + \frac{1}{2} - \Phi\left(\frac{15}{\sqrt{\frac{1500}{12}}}\right) =$$

$$= 1 - 2\Phi\left(\frac{15}{\sqrt{\frac{1500}{12}}}\right)$$

(b) $Y_n = \sum_{k=1}^{n} X_k$

$P\{|Y_n| < 10\} = 0.9$

$$2\Phi\left(\frac{10}{\sqrt{\frac{n}{12}}}\right) = 0.9 \Rightarrow n \approx 441$$

① A - obe izvučene loptice su crvene

B - među izvučenim lopticama je jedna crvena.

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{5}$$

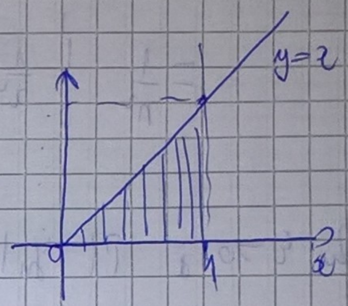
Mogućnosti: C_1C_2, C_1P, C_1B, C_2B, C_2P, BP

3. $\varphi_Y(y|x) = \begin{cases} \frac{1}{x}, & 0<y<x \\ 0, & \text{inače} \end{cases}$ $\qquad \varphi_X(x) = \begin{cases} 2x, & 0<x<1 \\ 0, & \text{inače} \end{cases}$

Odrediti $\varphi_X(x|y)$.

$$\varphi(x,y) = \varphi_X(x)\,\varphi_Y(y|x)$$

$$\Rightarrow \varphi_{(X,Y)}(x,y) = \begin{cases} 2, & 0<x<1,\ 0<y<x \\ 0, & \text{inače} \end{cases}$$



$\Rightarrow \varphi_Y(y) = \int_y^1 \varphi(x,y)\,dx$, za $0<y<1$

$\Rightarrow \varphi_Y(y) = \int_y^1 2\,dx = 2(1-y)$, $0<y<1$

Dalje je $\varphi(x,y) = \varphi_Y(y)\,\varphi_X(x|y)$, pa je za $0<y<1$

$$\varphi_X(x|y) = \frac{1}{1-y}, \quad \text{za} \quad y<x<1$$

$$\varphi_X(x|y) = 0, \quad \text{inače.}$$

4. $\left(\frac{1}{nX_n}\right)_{n\in\mathbb{N}}$, $X_1, X_2, \ldots$ nezavisne, $X_n : \mathcal{U}(0,1)$, $n=1,2,\ldots$

Neka je $Y_n = \frac{1}{nX_n}$

Tada je $F_{Y_n}(y) = P\{Y_n<y\} = P\{\frac{1}{nX_n}<y\} = P\{X_n > \frac{1}{ny}\} =$

$$= \begin{cases} 1-\frac{1}{ny}, & \frac{1}{n}<y<\infty \\ 0, & y\le \frac{1}{n} \end{cases} \xrightarrow[n\to\infty]{} \begin{cases} 1, & y>0 \\ 0, & y\le 0 \end{cases} = F_0(y)$$

$\Rightarrow Y_n \xrightarrow{r} Y\equiv 0$ (konvergira ka 0 u raspodeli)

Pošto je $Y\equiv 0$ konstanta $\Rightarrow$ konvergira u verovatnoći ka $Y\equiv 0$.

$E(Y_n^2) = ?$ (ispitujemo srednje kvadratnu konvergenciju)

$$E(Y_n^2) = \int_{-\infty}^{\infty} y^2 \varphi_{Y_n}(y)\,dy \qquad Y_n^2 = \frac{1}{n^2 X_n^2}$$

$$\Rightarrow E(Y_n^2) = E\left(\frac{1}{n^2} \frac{1}{X_n^2}\right) = \frac{1}{n^2} \int_{-\infty}^{\infty} \frac{1}{x^2} \varphi_{X_n}(x)\,dx =$$

$$= \frac{1}{n^2} \int_0^1 \frac{1}{x^2}\,dx = \infty$$ $\Rightarrow$ nemamo konvergenciju u srednje kvadratnom

Za $\varepsilon > 0$ i $P\{|Y_n| \geq \varepsilon\} = P\{X_n < \frac{1}{n\varepsilon}\} = \begin{cases} \frac{1}{n\varepsilon}, & n \geq \frac{1}{\varepsilon} \\ 1, & n < \frac{1}{\varepsilon} \end{cases}$

$\rightarrow \sum_{n=1}^{\infty} P\{|Y_n| \geq \varepsilon\}$ divergira

$\Rightarrow$ nemamo skoro sigurnu konvergenciju